A Path towards Strong Sustainability

Youngho Chang[♣]

December 2011

ABSTRACT

Sustainable development (SD) is a path that ensures the well-being of the present and future generations. There are two main concepts in the study of SD, namely weak sustainability (WS) and strong sustainability (SS). Two indicators of SD, Genuine Savings and Environmental Sustainability Index make use of WS and SS respectively. Due to the contrasting assumptions of WS and SS, the results from both sets of indicators contradict one another. Hence this study introduces the Hybrid Sustainability Model (HSM) where both concepts are used to redefine the path towards SD. First, the Solow-Hartwick sustainability model is extended to include technological and population change. This extension ensures that WS is achievable by using less than the total Hotelling rents. Second, SS could be ensured when the residual Hotelling rents is invested so as to maintain the stock of renewable natural resources towards a “Safe Minimum Standards” level. SD is then defined to be achieved in which conditions for both WS and SS are met.

Keywords: Hybrid Sustainability Model; Ecological services; Sustainable Development

JEL Classification: Q01, Q20, Q32, Q43

This is Research in Progress. Please do not cite.

[♣] Chang is with Division of Economics, School of Humanities and Social Sciences, Nanyang Technological University, 14 Nanyang Drive, Singapore 637332; Tel: +(65) 6316 8781; Email: isyhchang@ntu.edu.sg;

I. Introduction

There are diverse and seemingly conflicting concepts in sustainability as it is evident that studies based on different concepts yield contrasting results.. For example, the Environmental Sustainability Index[1] showed that 73 nations are unsustainable while the World Bank's ranking of sustainability showed that only 32 nations are unsustainable (Yale, 2005 and World Bank, 2006). The confusion is further compounded when nations such as Russia and Ecuador are shown sustainable on the Environmental Sustainability Index, but are considered unsustainable by the World Bank's ranking. The root of this confusion is because there is a lack of agreement on the path to take towards achieving SD.

The two main branches of SD are weak sustainability (WS) and strong sustainability (SS). Weak and strong sustainability approaches are fundamentally different as the former assumes that man-made capital is substitutable for natural resources, with the latter assuming otherwise. As such, proponents of either model are not agreeable with one another on how to achieve sustainable development. Currently, the WS approach is dominated by the Solow-Hartwick sustainability model and developments in this area are mainly to improve upon the original model. SS on the other hand, is not dominated by any models and there are numerous models that adopt the SS approach. Contrasting assumptions between WS and SS resulted in few attempts to reconcile the models of weak and strong sustainability, one of these few attempts were from Common and Perrings and their verdict is that WS is both insufficient and unnecessary for SD (Common and Perrings, 1992). However, there has been very little research following Common and Perrings' work as literatures on SD have developed separately and exclusively along the path of weak or strong sustainability. This paper seeks to use approaches from both weak and strong sustainability and find out if there exists a way to achieve both states so that there is a more agreeable path towards SD.

The rest of this paper is structured as follows. This paper revisits the Solow-Hartwick model (Solow; 1974 and 1986 and Hartwick; 1977, 1978a, and 1978b) and shows that an economy can achieve intergenerational equity by mandating the Hartwick rule of "investing all rents from natural capital into renewable capital". In section 2, it reviews the similarity and the differences in economic and ecological approach for sustainable development. It derives a more general form of the Hartwick rule by relaxing the assumptions of constant population and no technological progress in section 3. Using this result, it derives the Hybrid Sustainability Model investment rule and shows, using propositions and numerical simulation, how weak sustainability can be attained. It further extends the Hybrid Sustainability Model and explains how the residual Hotelling rents should be utilised in order to achieve strong sustainability. Lastly, section 4 concludes this study and presents directions for future research.

II. Economics and Ecological Approaches in Sustainable Development

2.1: Definition of Sustainable Development

'Sustainability' (use interchangeably with 'Sustainable Development') has received intense attention in the economics, ecological and political realms in recent decades. The Brundtland report[2] produced by the World Commission on Environment and Development (WCED) in 1987 is regarded as a breakthrough piece of work that propelled "sustainable development" into the political agendas of nations around the world (Perman, *et al*., 2003). The Brundtland report also contributes to the concept of Sustainable Development (SD hereupon) by defining SD as, "…development that meets the needs and aspirations of the present without compromising the ability to meet those of the future (WCED, 1987)".

While this definition reaches a consensus with most politicians and public, it is inadequate for policies measures due to the vagueness of the terms (Beckerman, 1994). For example, "needs and aspirations" are too broadly defined and more exact definitions are needed in order to have meaningful discussions. Owning to this vagueness, many disciplines offered their own interpretation of the Brundtland report. The following has been accepted to be the de facto definition of SD in economics, "…non-declining utility of a representative member of society for millennia into the future (Pezzey, 1992)" (Perman, *et al*., 2003; Neumayer, 2003). This definition though seems similar to that in the Brundtland report, actually irons out a lot of uncertainties. The term "needs and aspirations" is replaced with the more workable economic concept of "utility" and the term "future" replaced with "thousands of years".

Weak and Strong Sustainability

To achieve SD, economists still need to determine the inputs in the utility function and then work out a rule to ensure at least non-declining utility for a long finite period. Regarding the utility function, consumption has generally been used as the sole determinant in the utility function in microeconomics, though other variables such as "pollution" and "stock of renewable resources" can be included as well (Hartwick, 1990; Neumayer, 2003).[3] By neoclassical theory, the only way to ensure a non-declining consumption is to have a constant capital stock (Solow, 1974). These two premises constitute the concepts of weak sustainability and distinguish weak sustainability from strong sustainability.

---

[1] A score of less than 50 on the Environmental Sustainability Index is considered a un-sustainability.

[2] The Brundtland report is also known as the "Our Common Future" report.

[3] The inclusion of "pollution" serves to decrease utility.

Before the Brundtland report, neoclassical growth theory and resource economics had already presented models that are able to achieve intergenerational equity (Solow, 1974, 1986; Hartwick, 1977, 1978a, 1978b). Intergenerational equity can be paraphrased as SD because both Hartwick and Solow captured the two premises in their model by using consumption as a measure of utility and came up with a rule that ensures a constant capital stock. This model is known as “Solow-Hartwick sustainability” (Common and Perrings, 1992) and this approach is known in the literature as “weak sustainability” (Daly 1994). The main assumption in weak sustainability (WS hereupon) is that natural capital and man-made capital are substitutes for one another and each can readily replace one another. Another name for WS is economic sustainability.

Conversely, most ecologists and a few economists, such as Daly, Boulding and Georgescu-Roegen believe that natural capital and man-made capital are mainly complements or only marginally substitutable. Hence in their view, Solow-Hartwick sustainability cannot lead to SD as the former violates their assumption of poor substitutability between natural and physical capital. This line of thought is known as strong sustainability (SS hereupon). As one can imagine, SS is known as ecological sustainability.

It is important to state now that WS and SS are not different concepts (Hediger, 1999; Perman, *et al*., 2003). From their respective definitions, we can infer that WS is a necessary but not sufficient condition for SS. This is because WS concerns itself with keeping the aggregate stock of all capital constant, while SS is more concerned with keeping stock of each type of capital constant (Daly, 1994).[4]

2.2: Approaches of Sustainable Development

The assumption that sets WS and SS apart is that WS believes in a high substitution possibility between natural and physical capital, while SS believes in a low substitution possibility. To elaborate, “capital” refers to any stock that has economic productive means. “Natural capital” refers to ecological services provided by forests, fisheries, etc and minerals such as oil, coal, etc. “Man-made capital” consists of “physical capital” and “human capital”. “Physical capital” is machines, tools and buildings. “Human capital” is our knowledge or skills that are used for productive means. A further categorization of “natural capital” is “renewable resources” and “non-renewable resources”. This segregation is necessary because if all resources are “renewable”, then SD can be easily achieved since we will always have enough inputs for production.

2.2.1: Weak Sustainability Models

Solow-Hartwick sustainability

How intergenerational equity can be achieved was examined under different scenarios, and especially of interest to us is the economy with exhaustible resources (Solow; 1974 and 1986). An intuitive solution of keeping the total capital stock constant was provided following Solow’s paper (Hartwick; 1977, 1978a, and 1978b). So altogether this is known as the Solow-Hartwick sustainability model. A simple version of the Solow-Harwick sustainability model is written as follows.

The main criticisms of this model are twofold (Hanley, *et al*., 1995). First, consumption may not be the only input in the utility function. For example, if appreciation of aesthetic natural sceneries is included in utility functions, then non-declining consumption may not necessarily lead to non-declining utility. However, there are no compelling reasons for us to believe that appreciation of aesthetic natural sceneries should be included in the utility function as much as it should not be included. The second criticism is that natural and man-made capital may not be as substitutable as assumed in the model. This is especially true when we are referring to ecosystem services such as natural floodplains, oxygen cycle, etc. (Perman, *et al.*, 2003).

Genuine Savings

The ‘genuine savings’ (GS hereupon) model is a WS indicator that is based on the Solow-Hartwick sustainability model. This indicator shows whether a country is on the path of sustainable development (Pearce and Atkinson, 1993). Accordingly, a nation is sustainable if the sustainability index is positive and vice versa. As the GS model took roots from the Solow-Hartwick sustainability model, any strengths or weaknesses can be applied from there to here. But as Pearce and Atkinson’s empirical results omit the effects of trade on sustainability, this omission was made subject of criticisms (Martinez-Alier, 1995). But this flaw can easily be rectified by extending the Solow-Hartwick sustainability model to include trade (Hartwick, 1995). Another criticism is that it was not properly stated in the study on how the depreciation rate of natural capital should be estimated (Gutes, 1996).[5] But it must be added that the ease at which the other data required for this model is available increases the GS model attractiveness to policy makers.

Despite the criticisms, the GS model is useful as it is based on a theory with strong economics fundamentals. Hence the World Bank (World Bank, 2006) uses the GS method for their calculation of SD index among nations.[6]

---

[4] Ayres (1998) argued that both WS and SS are unrealistic as both concepts imply a centralized decision making process.

[5] Gutes (1995) also discussed the suitability of GS as a rule of thumb for SD.

[6] The World Bank uses a more complicated version of genuine savings than what is described in this paper.

2.2.2: Strong Sustainability Models

Unlike WS, there are no representative models of SS in current literature. We show a few models that individually emphasize different aspects of SS.

Non-declining natural capital stocks

The London school defines the SS assumption of non-substitutability between natural and man-made capital as the maintenance of our natural capital stock at a certain level (Pearce and Turner, 1990; Klaasen and Opschoor, 1991). The rule in this approach is that natural capital stock must be maintained at an ecologically determined level where it will not be breached under any conditions. This strict condition is loosened in the similar “safe minimum standards” (SMS hereupon) approach (Ciriacy-Wantrup, 1952; Bishop, 1978 and 1993). SMS approach allows the natural capital level to be breached beyond the critical level when the opportunity cost of not utilizing these resources is too large. These two models have the advantage of being simple to explain and understand and hence should be more easily included into political agendas. However, they may not work for two reasons. First, minerals such as copper, tin, oil, etc. are all non-renewable natural resources and play an invaluable part in the progress of our economy. If we are not allowed to mine for new supplies, there can be detrimental effects to the economy if we do not have immediate substitutes for these resources. Second, it is not stated in the SMS approach on how the opportunity costs are to be identified. This ambiguity creates difficulty when devising policies to carry out these approaches.

Daly’s Operational Principles

As an alternative to the non-declining capital stock approach, “operational principles” (OPs hereupon) are a collection of prescriptions leading to SD applicable for any economy (Daly, 1990). OPs can be categorized according to the different factors of production in the economy. First, for the management of renewable resources, harvests should not exceed the regenerative rate and pollution is only allowed till the maximum of nature’s assimilative capacity. Second, for natural and man-made capital, both capital stocks should be maintained at an “optimal level”. This “optimal level” is defined as the level that gives maximum yield per time period for natural resources. While the “optimal level” for man-made capital is an arbitrary level depending on the production function.[7] This follows the belief of Daly that man-made capital and natural capital are essentially complements. Third, for non-renewable resources, part of the net receipts should be invested into a renewable substitute at a rate such that by the time the non-renewable resource is economically depleted, the substitute can fully assume the former role.[8] Lastly, for technology, we should focus on technology that reduces resources throughput.

As with other models of SS, OPs are easier to comprehend when compared to WS models. Another advantage is that Daly’s OPs are much more comprehensive than the SMS model since it covers the treatment of almost all forms of resources. But upon closer examination, an inconsistency of OPs is that while on one hand, man-made capital and natural capital are assumed to be complements, but on the other, it allows renewable resources to substitute for non-renewable resources. Another criticism is its operationability (Hanley, *et al*., 1997, 432). Information such as the assimilative capacities of the environment is not known and quite possibly dynamic and changing over time. It is not sure if such information can ever be found, let alone devise policies to carry it out.

Common-Perrings Model

The Common and Perrings model (1992) can be considered to be the first to combine economic sustainability (WS) and ecological sustainability (SS). In their model, they combined Hartwick rule with Holling’s resilience to provide a better understanding of SD across the disciplines of economics and ecology. While the Hartwick rule is an epitome of WS, Holling’s resilience does not share the same status in SS (Common and Perrings, 1992). Their rationale in choosing Holling’s resilience lies in that this concept can be readily expressed in an economic framework, unlike other SS models such as OP. Common and Perrings’ model concludes that economic sustainability is neither sufficient nor necessary for ecological sustainability. To achieve the latter, they suggested that new approaches are needed in economics. But as shown, WS is a necessary condition for SS. Their different result must be in part due to the usage of Holling’s resilience as their definitive model for SS. Hence Common and Perrings conclusion can only be accepted on the assumption that Holling’s resilience is representative of SS.

2.2.3: Hybrid Approach for Sustainability

WS model faces the difficulties of justifying the assumption of a high substitution possibility between natural and man-made capital, while SS models are difficult to operate in reality. In view of these, there is a need to introduce a new indicator of sustainability that combines both WS and SS. Areas such as the production of output is dealt using neoclassical

---

[7] For example, if the production function is $Y=min(aR, bK)$, then K should be maintained at aR/b.

[8] Economic depletion is reached when 80-percent of known reserves are harvested.

equimarginal principles while areas such as renewable natural resources should be dealt by SS models such as “safe minimum standards” as externalities render market failure (Toman, 1992).[9]

To achieve SD, the stock of non-renewable natural and man-made capital must jointly be held constant, while the stock of renewable natural resources that provide renewable ecological good and services must be held at least at the SMS level.

2.3: Implications of Different Approaches in Sustainable Development

Following the 1987 Brundtland Report, a host of indicators for SD appeared. Some of these indicators are based on SS assumptions, such as the “Environmental Sustainability Index” (ESI hereupon) (Yale, 2005), genuine progress indicators, and some are based on WS framework, such as “Genuine Savings” (GS hereupon) (World Bank, 2006). In the latest rankings, both GS and ESI have different environmental ‘winners’ and ‘losers’. For example, South Korea is ranked 122nd on the ESI, but has a GS rate of 23.6%, higher than even Finland, the ESI leader. Many such examples can be found when comparing these two sets of SD indicators. These contrasts thus form the basis of the research objective. This study examines how an economy that is presently in a weakly sustainable state can reach a more desirable sustainable state. The desired sustainable state is not defined to be the state of SS mainly due to two reasons. First, any attempts to reach SS from WS force us to give up the assumptions in WS. This is untenable as WS and SS contradicts each other in their respective definition of substitution possibilities between natural and man-made capital. Second, SS is a restrictive condition that may impede economic development if followed strictly (Hanley, *et al.*, 1997). Therefore economic growth may be sacrificed in the process if we are able to achieve SS. In light of these reasons, the chosen sustainable state is the hybrid sustainable state. This state is chosen because it represents concepts found in both WS and SS, thus we can readily apply the model with neoclassical assumptions and yet tackle issues that are not addressed using economic means.[10] Further, the conditions are not as restrictive as those in SS models and so there is a less likelihood of impeding economic progress amidst the pursuit of SD.

III. Solow-Hartwick and Hybrid Sustainability Model

3-1 Model with Population Growth and Constant Technology[11]

The Hartwick rule is a particular case whereby there are no technological progress and constant population. Hence the result of “invest all rents from natural capital into renewable capital” only suffices in illustrating the usefulness of the Hartwick rule in this type of economy. A more general version of the Hartwick rule is needed to explain for the increasing per capita consumption the world has experienced (Maddison, 1995). This section explores this possibility in various settings of economy such as an economy with increasing population, an economy with technological progress, and an economy with both technological progress and population change.

We suppose that an economy has the production function,

$$Y = K^{\alpha} R^{\beta} L^{1-\alpha-\beta}, \tag{1}$$

where Y is the output, K is the renewable capital, L is the labour, R is the non-renewable natural resource and α and β are the parameters that represent renewable capital and non-renewable natural resource share of the production respectively. All factor inputs are well-behaved, such that $\frac{\partial Y}{\partial X} \equiv Y_X > 0$, $\frac{\partial^2 Y}{\partial X^2} \equiv Y_{XX} < 0$ and $X \in (K, R, L)$ where $\frac{\partial Y}{\partial X}$ represents partial derivative of X with respect to Y and $\frac{\partial^2 Y}{\partial X^2}$ represents second order derivative. We assume that the variables in this economy exhibits quasi-arithmetic growth throughout the time period examined such that,

$$Z(t) = Z(0)(1+\mu t)^{g_Z}, \tag{2}$$

where $Z = (Y, C, L, K, R)$. $Z(t)$ refers to the amount or stock of the variable at time t. μ is the parameter that is identical for all variables and is greater than 0 ($\mu > 0$). g_Z indicates whether the variable is increasing or decreasing and μ indicates the limits of the economy.

---

[9] The Equimarginal Principle refers the situation when marginal benefit equals marginal cost.

[10] The assumptions are here are that technological progress is exogenous and there is high substitutability between man-made capital and non-renewable resources (Solow, 1957 and Asheim, 2003).

[11] This following section is based on Asheim, *et al*. (2007).

Lastly, the rate of change for any variable is obtained when we take natural log from both sides of equation (2) and differentiate both sides with respect to time:

$$\frac{\dot{Z}(t)}{Z(t)} = \frac{g_Z \mu}{(1+\mu t)} \tag{3}$$

Deriving per capita consumption

Total investment is also the total savings in the economy such that $sY = I$, where I is the total investment in man-made capital and s is the proportion of output that is saved. sY is also known as the gross investment. The net investment, s_nY is known as,

$$s_n Y = sY - nK \tag{4}$$

where n is the rate of population growth such that $n = \frac{g_n \mu}{(1+\mu t)}$, and s_n is the net proportion of savings that is invested in capital after we account for the growth in population.

This model gives the amount of required investment that must be undertaken to make up for the depletion in non-renewable resources in order to maintain a constant per capita output. To rewrite the output function in per capita notations:

$$y = k^{\alpha} r^{\beta} \tag{5}$$

By taking natural log from both sides of equation (5) and differentiating both sides with respect to time, we have an expression of the rate of change of output,

$$\frac{\dot{y}}{y} = \frac{\alpha \dot{k}}{k} + \frac{\beta \dot{r}}{r} \tag{6}$$

We substitute a variant of the Hotelling rule in equation (7) into equation (6).

$$\frac{\dot{y}}{y} - \frac{\dot{r}}{r} = \frac{\alpha y}{k} \tag{7}$$

The left hand side of equation (7) is the rate of change of net rent of non-renewable resources. We can now eliminate $\frac{\dot{r}}{r}$ using equation (6) and (7),

$$\frac{k}{y}\left(\frac{\dot{k}}{k} - \frac{\dot{y}}{y}\right) = \frac{(1-\alpha-\beta)\frac{\dot{k}}{y} + \alpha\beta}{1-\beta} \tag{8}$$

where the left hand side of equation (8) is the change of the capital-output ratio, (i.e. $\dot{\left(\frac{k}{y}\right)}$) and $\frac{\dot{k}}{y}$ is the net savings rate, s_n.

By denoting $\frac{k}{y} = x$, we rewrite equation (8) as:

$$\dot{x} = \frac{(1-\alpha-\beta)s_n + \alpha\beta}{1-\beta} \tag{9}$$

Integrating equation (9) with respect to time, we obtain an expression for the capital-output ratio.

$$x(t) = x(0) + \sigma t \tag{10}$$

where

$$\sigma = \left[\frac{(1-\alpha-\beta)s_n + \alpha\beta}{1-\beta}\right] \tag{11}$$

and $x(0)$ is the capital-output ratio at time zero. Equation (10) suggests that $x(t)$ is a linear function of time as σ is a parameter. By equation (2), g_Z is unity if $Z(t)$ is a linear function, hence by the same analogue,

$$x(t) = x(0)(1+\mu t) \tag{12}$$

Equation (12) is used to determine the growth rate of output and consumption.

Per capita output, y can be defined as $\frac{k}{x}$ and we express the growth rate of per capita output as:

$$\frac{\dot{y}}{y} = \frac{s_n - \sigma}{x} \tag{13}$$

Recall that all variables are assumed to follow a quasi-arithmetic function and using equation (3), the growth rate of the per capita output is:

$$\frac{\dot{y}(t)}{y(t)} = \frac{g_y \mu}{(1+ut)}. \tag{14}$$

We solve for the per capita output growth rate, g_y by eliminating $\frac{\dot{y}}{y}$ using equation (13) and (14). The growth rate of per capita output is $\left(g_y = \frac{(s_n - \sigma)(1+\mu t)}{x\mu} \right)$, the right hand side expression is further simplified using equation (10) and (12) and we have:[12]

$$g_y = \frac{s_n}{\sigma} - 1. \tag{15}$$

The per capita output and per capita consumption functions are hence given by,

$$y(t) = y(0)(1+\mu t)^{\frac{s_n}{\sigma} - 1} \tag{16}$$

$$c(t) = (1-s)c(0)(1+\mu t)^{\frac{s_n}{\sigma} - 1} \tag{17}$$

Investment rule for constant per capita output

The objective is to derive a general investment rule for per capita output in the face of population growth. If we take the time derivative of equation (16), we obtain $\dot{y}(t) = y(0)\left(\frac{s_n}{\sigma} - 1\right)\mu(1+\mu)^{\left(\frac{s_n}{\sigma} - 2\right)}$. Since $y(0), \mu, t, s_n, \text{and } \sigma$ are positive, the only way for a constant per capita output, $(\dot{y}(t) = 0)$, is when $\left(\frac{s_n}{\sigma} = 1\right)$, which implies $(s_n = \sigma)$. Substitute $\left(s = s_n + n\frac{K}{Y} \right)$ from equation (11) and arrange the gross savings rate, s to the left hand side,

$$s = n\frac{K}{Y} + \beta. \tag{18}$$

Equation (18) is not yet a closed-form solution, but we shall leave it as it is now and undertake further simplification in the following section where we consider technological progress.

---

[12] See Appendix, Proof (3) for derivation of equation (15).

3-2 Model with Technological Progress and Constant Population[13]

Suppose now that the economy has quasi-arithmetic technological progress but has a constant population (i.e., $\frac{\dot{L}}{L} = 0$) and then the production function is

$$Y = AK^{\alpha} R^{\beta} L^{1-\alpha-\beta}, \tag{1'}$$

where *A* represents the level of technology in the economy and the other specifications are the same as that in section 3-1. Note that the gross and net savings rates in equation (4) are the same when there is no growth in population. But we choose to define savings as s_n. This is because s_n is the amount of gross savings left when we have accounted for the increase in population. Hence, the savings in an economy with constant population is related closer to s_n rather than *s* by definition even though both are mathematically identical. Also, since population is constant in this model, there is no distinction between the growth rates of *Y*, *K*, *R*, etc. in per capita and gross term.

Deriving of per capita consumption

Following the same steps, we start off with the rate of change of output by taking natural log of equation (1') and differentiating with respect to time,

$$\frac{\dot{Y}}{Y} = g_A \frac{\mu}{1+\mu t} + \alpha \frac{\dot{K}}{K} + \beta \frac{\dot{R}}{R} \tag{19}$$

Before we proceed with the derivation of the growth rate of the capital-output ratio $\dot{k}$, it is important to note that the term $\left(\frac{\mu}{1+\mu t}\right)$ in equation (19) is identical to the grow rate of capital-output ratio, $\left(\frac{\dot{K}}{K} - \frac{\dot{Y}}{Y}\right)$. This insight is useful as it helps to express $\dot{k}$ in more familiar notations. Hence by eliminating $\frac{\dot{R}}{R}$ using equation (7) and (19), we have,

$$x(t) = x(0) + \sigma t \tag{20}$$

where

$$\sigma = \left[\frac{(1-\alpha-\beta)s_n + \alpha\beta}{1-\beta+g_A}\right] \tag{21}$$

The output and consumption per capita is hence similar to that in section 3.1 following equation (14) and (15) respectively:

$$y(t) = y(0)(1+\mu t)^{\frac{s_n}{\sigma}-1} \tag{15'}$$

$$c(t) = (1-s)c(0)(1+\mu t)^{\frac{s_n}{\sigma}-1} \tag{16'}$$

Investment rule for constant per capita output

We can infer from equation (15') that to have a constant per capita output, the rate of savings, *s* must be equal to the constant term, σ, i.e $\left(s_n = \frac{(1-\alpha-\beta)s_n + \alpha\beta}{1-\beta+g_A}\right)$. We obtain the investment rule by arranging the rate of savings, s_n to the left hand side,

$$s_n = \frac{\alpha\beta}{g_A + \alpha} \tag{22}$$

Therefore, this is the investment rule to maintain a constant output per capita with technological progress and constant population.

---

[13] The following section is based on Asheim, *et al.* (2005).

3-3 Hybrid Sustainability Model with Technological and Population Growth

To depict reality more accurately, we combine the investment rule in equation (18) and (22) to obtain a new rule that applies to economies that experience changes in both their technology and population. The new investment rule is obtained when we substitute $\left(s_n = s - n\dfrac{K}{Y}\right)$ into equation (22).

$$s = \frac{\alpha\beta}{g_A + \alpha} + n\frac{K}{Y} \tag{23}$$

Equation (23) is not yet a closed-form solution since output, *Y* remains in the equation. To remove *Y*, multiply both sides of equation (30) by *Y* and $\dfrac{R}{R}$, we have,

$$sY = \left(\frac{\alpha}{g_A + \alpha}\right)\frac{\beta Y}{R}R + nK \tag{24}$$

Since $\left(\dfrac{\beta Y}{R}\right)$ is the Hotelling rent for a unit of non-renewable resource, equation (24) can be expressed in terms of the total Hotelling rents received so as to compare better with the Hartwick rule. We have the investment rule for the Hybrid Sustainability Model,

$$sY = \left(\frac{\alpha}{g_A + \alpha}\right)(Y_R - a)R + nK \tag{25'}$$

Interpretation of the HSM investment rule

We now consider whether the required investment to keep per capita output constant is greater or lesser than the entire Hotelling rents under different permutations of technological and population growth rates.

Proposition: *If a country has positive population growth and technological progress,* $(n, g_A > 0)$ *such that the population growth-augmented level of capital is larger than the technological progress-augmented level of the Hotelling rents* $\left[nK > \left(\dfrac{g_A}{g_A + \alpha}\right)(Y_R - a)R\right]$, *then the amount of investment in man-made capital needed to maintain per capita output at the current period level is more than the entire Hotelling rents.*

Proof: Using equation (25'), in the situation of positive population growth and technological progress. The required investment is $\left[sY = nK + \left(\dfrac{\alpha}{g_A + \alpha}\right)(Y_R - a)R\right]$, but with the condition $\left[nK > \left(\dfrac{g_A}{g_A + \alpha}\right)(Y_R - a)R\right]$, the required investment to keep per capita output constant is more than the entire Hotelling rents. (QED)

The condition, $\left[nK > \left(\dfrac{g_A}{g_A + \alpha}\right)(Y_R - a)R\right]$ in the Proposition implies that the required investment is more likely to be greater than the Hotelling rents when the amount of man-made capital is larger than the Hotelling rent and when technological progress is slow. Section 3.4 examines if the Proposition still holds for plausible amount of man-made capital and level of technological progress.

In a nutshell, the amount of required investment varies according to the population and technological growth rates. Higher population growth rate increases the amount of required investment while faster technological progress decreases the amount of required investment. The HSM investment rule is more general than the Hartwick rule and hence can be applied to more countries. This rule is also different from the ones derived in Asheim, *el at*. (2005) as the cases of population growth and technological progress are considered separately and not together.

3-4 Numerical Simulation of the HSM Investment Rule

Different permutations of technological progress and population growth rate are considered in previous section. A numerical simulation is conducted to assert the empirical viability of the Proposition. Before we carry on with the simulation, some estimations and assumptions about the variables are needed. The capital share of output, α is taken to be 0.33 as estimated by Baier, *et al*. (2002). Rate of technological progress is defined as Total Factor Productivity (TFP) or also known as "Solow Residual" (Solow, 1957). TFP measures the per capita growth in output that is not attributed to growth in of other inputs, such as labour or capital (Baier, *et al*., 2002).[14] But the maximum potential technological progress rate, g_A is used in equation (25') rather than the TFP, $\left(\frac{g_A\mu}{1+\mu t}\right)$. Since g_A is not readily available, thus TFP is used to estimate for g_A. The growth rate of population, *n* is defined as the instantaneous growth rate or known as the intrinsic growth rate,

$$N_{1990} = N_{1900}e^{nt} \tag{25}$$

where N_{1990} and N_{1900} represent population in year 1990 and 1900 respectively, while *e* is the base of natural logarithm and *t* is the time period. The Hotelling rents represent the price of non-renewable resources multiplied by the units of non-renewable resources used rather than extracted. This is because countries such as Singapore or Japan imports almost of all their non-renewable resources and net importer countries need to invest more in man-made capital so as to make up for the terms of trade effect in the future (Asheim, 1986; Hartwick, 1995).

With all the terms defined, a numerical simulation of the HSM investment rule is presented in Table 3.1.

Table 3.1: NUMERICAL SIMULATION OF REQUIRED INVESTMENT

No.	(a) α	(b) Technological progress, g_A	(c) Population Growth rate, *n*	(d) Man-made capital, *K*	(e) Hotelling rents, $(Y_R - a)R$	(f) sY
(1)	0.33	0.05 (High)	0.011	50,000	30,000	26602.63
(2)	0.33	0.03 (Medium)	0.011	50,000	30,000	28050.00
(3)	0.33	0.01 (Low)	0.011	50,000	30,000	29667.64
(4)	0.33	0.007	0.03	31,795	46,667	46,651.5

Row (4) data indicates Dubai's data in million Dhs for year 2003. Row (4), column (a) is obtained from (Baier, *et al*., 2002). Row (4), columns (b) and (c) is obtained from (Rettab and Kwaak, 2005). Row (4), columns (d) and (e) are obtained and/or computed from Dubai Statistical Yearbook (2004).

Technological progress takes on high, medium or low value. The population growth rate is taken to be the world population growth rate which is 0.011 or 1.1% (U.S. Census Bureau website, 2005). Man-made capital is deliberately chosen to be larger than the Hotelling rents as this provides a stricter case for the required investment to be smaller than the Hotelling rents.

In row (1), (2), and (3) of Table 3.1, the amount of required investment is lesser than the Hotelling rents but the difference becomes smaller as the rate of technological progress slows down. This shows that required investment is generally lesser than the entire Hotelling rents for plausible levels of technological progress and man-made capital. Therefore, while the Proposition is mathematically valid, it may not be applicable to real cases. To confirm this result, we calculate the required investment for a real-life economy with positive technological progress and population growth. Dubai is chosen because of the availability of data and because the principality had experienced technological progress and high population growth. From row (4), the required investment for Dubai is 46.651.5 million Dhs which is only negligibly lesser as compared to its Hotelling rents of 46,667 million Dhs. This is due to Dubai's slow rate of technological progress and very high rate of population growth. Nonetheless, as Dubai ascends closer to the ranks of developed economies, their required investment is bound to decrease as their level of technology increases more rapidly and population growth slows down.

The HSM investment rule has shown that in cases of positive population growth and technological progress, economies are generally "more" than weakly sustainable as they still have residual Hotelling rents left after ensuring for constant per capita output from the previous period. In the next section, the discussion is on how the usage of the residual Hotelling rents can lead to strong sustainability.

3-5 Putting Strong Sustainability to HSM

Strong Sustainability in the Hybrid Sustainability Model

WS alone cannot ensure for sustainable development (SD) due to markets' undervaluation of ecological goods and services. Hence SS approaches are needed in the pursuit of SD. Several models of SS were discussed and the HSM adopts the Safe Minimum Standards (SMS) approach as its SS definition. However, unlike the original SMS model, the HSM does not

---

[14] Barro *et al*. (2003) has a detailed treatment on the derivation of TFP using Solow growth frameworks.

seek to maintain all stocks of natural resources at the SMS level. This is because non-renewable resources are assumed to be substitutable by man-made capital, and hence the HSM focuses on only maintaining the stock of renewable natural resources towards the SMS level. By maintaining and/or conserving renewable natural resources at the SMS level, the HSM ensures that the next generation can enjoy the same ecological goods and services that the current generation are enjoying. In notational form we have the time derivative of the stock of renewable natural resources as follows,

$$\begin{aligned} &\dot{S}_{Rr} = g(S_{Rr}, I_{Rr}) - R_{Rr} \\ &\text{s.t. } S_{Rr} \geq \bar{S}_{Rr} \end{aligned} \tag{26}$$

where S_{Rr} is the stock of renewable natural resources, $g(S_{Rr}, I_{Rr})$ is the growth rate of this stock, I_{Rr} is the remaining Hotelling rents and R_{Rr} is the amount of renewable resources harvested. The stock of renewable natural resources is assumed to be depleted only through human activities such as harvesting and natural occurrences such as volcanic eruptions.[15] But the latter is of little concern as we virtually cannot avoid natural disasters. The growth rate of renewable natural resources, $g(S_{Rr}, I_{Rr})$ is an increasing function of its existing stock and investment and it follows a logistics growth path so that its growth rate is dependent upon the size of its stock. If the existing stock is depleted or polluted such that it goes below the SMS level of $\bar{S}_{Rr}$, the stock of resources will fail to renew itself (Ciriacy-Wantrup, 1952; Shaffer, 1981). Therefore, to ensure SS in the HSM, the residual Hotelling rents must be invested in a way to ensure that the stock of renewable natural resources remains above $\bar{S}_{Rr}$.

IV. Conclusion

This study revisits the Solow-Hartwick model of sustainability and introduces the Hybrid Sustainability Model (HSM) with the aim of incorporating both weak sustainability (WS) and strong sustainability (SS) approaches in a single model so as to provide a less contentious path for Sustainable Development (SD). The motivation behind the HSM is to resolve the confusion surrounding the path towards SD due to the differing assumptions of WS and SS. The most interesting result of the HSM investment rule is that economies with positive population growth and technological progress are generally ‘more’ than weakly sustainable. This tells us that most economies need not use all of their Hotelling rents to ensure for WS.

The HSM also suggests that the residual Hotelling rents must be invested in renewable natural resources towards the Safe Minimum Standards (SMS) level to achieve SS. The focus is on renewable natural resources because ecological goods and services are not used in a sustainable manner due to their common resources and pure public goods characteristics (Dasgupta, *et al.*, 2000; Hardin, 1968). This is in direct contrast to the Solow-Hartwick model where renewable natural resources are believed to be used in a sustainable manner (Hartwick and Olewiler, 1998). To make up for this undervaluation and at the same time achieve SS, the HSM suggests that the residual Hotelling rents are to be invested in renewable natural resources towards the SMS level. Together, when these two conditions of WS and SS are met, SD could be achievable.

An insight from the HSM is that WS and SS are both individually necessary but insufficient conditions for SD. This follows that, while WS ensures a non-declining per capital output, no solutions are offered for the over-consumption of renewable natural resources. On the other hand, while SS ensures that renewable natural resources are not depleted beyond the SMS level, it does not ensure a constant per capita output nor does it suggest a source for the funds to maintain the stock renewable natural resources. It is more likely that SD can be achieved when economies follow both paths of WS and SS, rather than exclusively down any individual path.

The viability of the HSM can be further asserted by future research that could provide an operational definition of the SMS, including the effects of pollution in the production function, and empirically assess the effectiveness or adequacy of investing the residual Hotelling rents in renewable natural resources. This adequacy is not simply measured by calculating the difference in the true and market valuation of ecological goods and services. Rather it should be measured by the amount needed to invest in the renewable natural resources that provides for these ecological goods and services.

---

[15] Although animals can also degrade the environment as in the case of reindeers on St. Matthew island, but the cause is ultimately human-induced as the reindeers were introduced as non-native species on the island (Miller, 2005).

References:

Ashiem, G. (1986) “Hartwick’s Rule in Open Economies.” *Canadian Journal of Economics*, Vol. 19, No. 3, 395-402.

Ashiem, G., W. Buchholz, J. Hartwick, T. Mitra, and C. Withagen (2005) “Constant Saving Rates and Quasi-Arithmetic Population Growth Under Exhaustible Resources Constraints.” *CESIFO* Working Paper No. 1573.

Asheim, G., B. Buchholz, and C. Withagen (2003) “The Hartwick Rule: Myths and Facts.” *Enviromental and Resource Economics*, Vol. 25, No. 2, 129-50.

Baier S., G. Dwyer Jr., and R. Tamura (2002) “How Important Are Capital and Total Factor Productivity for Economic Growth?” Working Paper *2002-2a*.

Barro, R. and X. Sala-I-Martin (2003) “*Economic Growth, 2nd edition*.” Cambridge: MIT Press.

Chichilnisky, G. and G. Heal, (1998) “Economic Return From the Biosphere.” *Nature*, Vol. 391, No. 6668, 629-30.

Chomitz, K., E. Brenes, and L. Constantino (1998) “Financing Environmental Services: The Costa Rican Experience and its Implications.” *Working Paper No. 20014*. Washington: World Bank.

Christensen, N., A. Bartuska, J. Brown, S. Carpenter, C. D'Antonio, R. Francis, J. Franklin, J. MacMahon, R. Noss, D. Parsons, C. Peterson, M. Turner, R. Woodmansee (1996) “The Report of Ecological Society of America Committee on the Scientific Basis for Ecosystem Management.” *Ecological Applications*, Vol. 6, No. 3, 665-91.

Ciriacy-Wantrup, S.V. (1952) “*Resource Conservation, Economics, and Policies*.” Berkeley: University of California Press.

Cohen, J. and D. Tillman (1996) “Biosphere 2 and Biodiversity: The Lessons So Far.” *Science*, Vol. 274. No. 5290, 1150-1.

Common, M. and C. Perrings (1992) “Towards an Ecological Economics of Sustainability”. *Ecological Economics*, Vol. 6, Iss. 1, 7-34.

Cornes, R. (1996) “*The Theory of Externalities, Public Goods, Club Goods, 2nd Edition*.” Cambridge, England: Cambridge University Press.

Costanza, R. and C. H. Perrings (1990) “A Flexible Assurance Bonding System for Improved Environmental Management.” *Ecological Economics* Vol. 2, 57-76.

Costanza, R. and Cornwell, L. (1992) “The 4P Approach to Dealing with Scientific Uncertainty.” *Environment* Vol. 34, No. 9, 12-21, 42

Costanza, R. and H. Daly (1992) “Natural Capital and Sustainable Development.”*Conservation Biology*, Vol. 6, No. 1, 37-46.

Costanza, R., R. d’Arge, R. de Groot, S. Farber, M. Grasso, B. Hannon, K. Limburg, S. Naeem, R.V. O’Neill, J. Paruelo, R.G. Raskin, P. Sutton, and M. van den Belt (1997) “The Value of the World’s Ecosystem Services and Natural Capital.” *Ecological Economics*, Vol. 25, Iss. 1, 3-15.

Daily, G. (1997) “*Nature’s Services.*” Washington D.C.: Island Press.

Dasgupta, P., S. Levin, and J. Lubchenco (2000) “Economic Pathways to Ecological Sustainability.” *BioScience*, Vol. 50, No. 4, 339-45.

Dubai Statistical Yearbook 2004, Dubai Municipality.

Hardin, G. (1968) “The Tragedy of the Commons.” *Science*, Vol. 162, No. 3859, 1243-8.

Hartiwck, J. (1977) “Investment of Rents from Exhaustible Resources and Intergenerational Equity.” *American Economic Review*, Vol. 67, No. 5, 972-4.

Hartiwck, J. (1978a) “Substitution Among Exhaustible Resources and Intergenerational Equity.” *Review of Economic Studies*, Vol. 45, No. 2, 347-54.

Hartiwck, J. (1978b) “Investment of Returns from Depleting Stocks of Renewable Resources and Intergenerational Equity.” *Economics Letters*, Vol. 1, No. 1, 85-8.

Hartwick, J. (1995) “Constant Consumption Paths in Open Economies with Exhaustible Resources”. *Review of International Economics*, Vol. 3, No. 3, 275-83.

Hartwick, J. and N. Olewiler (1998) “*The Economics of Natural Resource Use, 2nd edition*.” New York: Addison-Wesley.

Hediger, W. (1999) “Reconciling “Weak” and “Strong” Sustainability”. *International Journal of Social Economics*, Vol. 26, Iss. 7/8/9, 1120-44.

Holl, K.D. and R.B. Howarth (2000) “Paying for restoration.” *Restoration Ecology* Vol. 8, 260-267.

Hotelling, H. (1931) “The Economics of Exhaustible Resources” *The Journal of Political Economy*, Vol. 39, No. 2, 137-75.

Howarth, R. (1997) “Sustainability as Opportunity.” *Land Economics*, Vol. 73, No. 4, 569-79.

Howarth, R. and S. Farber (2002) “Accounting for the Value of Ecosystem Services.” *Ecological Economics*, Vol. 41, Iss. 3, 421-9.

Loomis, J., P. Kent, L. Strange, K. Fausch, and A. Covich (2000) “Measuring the Total Economic Value of Restoring Ecosystem Services in an Impaired River Basin: Results from a Contingent Valuation Survey.” *Ecological Economics*, Vol. 33, Iss. 1, 103-17.

Millennium Ecosystem Assessment, http://www.millenniumassessment.org/en/index.aspx, Last accessed: 31 March 2006.

Miller, G. (2005) “*Living in the environment: principles, connections, and solutions, 14th edition*.” Pacific Grove CA: Thomson Brooks/Cole.

Murtough, G., Aretino, B., and Matysek, A (2002) “*Creating Markets for Ecosystem Services/*” *Productivity Commission Staff Research Paper*, AusInfo, Canberra.

Pagiola, S. (1996) “*Republic of Croatia Coastal Forest Reconstruction and Protection Project*.” Report No. 15518-HR. Washington: World Bank.

Palm, R (1990) “*Natural Hazards: An Integrative Framework for Research and Planning*.” Baltimore: Johns Hopkins University Press.

Postel, S. and B. Thompson, Jr. (2005) “Watershed Protection: Capturing the Benefits of Nature’s Water Supply Services.” *Natural Resources Forum*, Vol. 29, Issue. 1, 98-108.

Rettab, B. And T. Kwaak (2005) “The Structure of Economic Development in UAE: Benchmarking Dubai against the other Emirates.” *DCCI*.

Schou, Poul (2000) “Polluting Non-renewable resources and Growth,” *Environmental and Resource Economics*, Vol. 16, 211-227.

Shaffer, M. (1981) “Minimum Population Sizes for Species Conservation.” *BioScience*, Vol. 31, No. 2, 131-4.

Smulders, S. and R. Gradus (1996) “Pollution Abatement and Long-term Growth.” *European Journal of Political Economy*, Vol. 12, Iss. 3, 505-32.

Solow, R. (1957) “Technical Change and the Aggregate Production Function.” *Review of Economics and Statistics*, Vol. 39, No. 4, 312-20.

Solow, R. (1974) “Intergenerational Equity and Exhaustible Resources.” *Review of Economics Studies*, Symposium, 29-46.

Solow, R. (1986) “On the Intergenerational Allocation of Natural Resources.” *Scandinavian Journal of Economics*, Vol. 88, Iss. 1, 141-9.

U.S. Census Bureau, International Programs Center (IPC), http://www.census.gov/ipc/www. Last accessed: 31 March 2006.

World Bank (2006) *Where is the Wealth of Nations*, Washington: World Bank.

Yale Center for Environmental Law and Policy (2005) *2005 Environmental Sustainability Index*, New Haven.