Real Option Analysis on Optimal Reserve Management for Strategic Resources

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Abstract

During 2000s, the demand for rare metals in industry fields has been increased by unstable international situations and resource nationalism. However, competitions for the resource security are being fiercing due to the finite rare metals and regional concentration of them, so the efficient reserve management considering the energy security is imminent now. This paper studies the optimal management of international price volatility and reserve resource using a real option anlaysis. Through the optimal price calculation for rare metals, the release time of reserve resource is reviewed and the cost effect for early provision of them can be identified. As a result, the price of analysed four rare metals has not reached the optimal value for release by government and when early reserve is performed, government can reduce the cost for resource reserve. By these results, we can find the economic incentive for early reserve of rare metals.

Key words: rare metals, reserve management, optimal price, real option

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1. Introduction

According to the increasing demand of IT, aviation and steel industry, the problem of demad and supply of rare metals which are used as essential raw materials in these industries is being recently noted. Especially, since 2000, rapid economic development of newly industrialized countries like China and India and investments expansion phenomenon for green industry primarily lead by developed countries aroused sudden increases of demand and suppply of high-tech product, by this, rare metals used as raw materials of high-tech product are being considered as important materials in the primary industries.

The world average consumption of rare metals from 2003 to 2007 approximately doubled. For examples, since 2000, the consumption of lithium is growing by annual average 6%. The demand of rare metals rapidly also increased from 3.25 billion dollars in 2002 to 12.59 billion dollars in 2008 in Korea, according to the development of high-tech industries.

The rare metals being used as critical components in industry development have the circumstance with limited suppliers. The reserves of these are relatively smaller than others due to the intrinsic charateristics of rare metals and the production is being performed in only some countries. In the rare-earth elements having relatively much reserves, China is producing more than 95% of world production of these due to payability problems. In the case of Korea and Japan, the primary producer of high-tech product, the demand for rare metals is very high, but both of them have supply shortage.

In addition to this, China having the richest volume of rare metals in the world is arousing the price increasing of these through their resource-nationalism policy and rigorous supervising for mining or exporting of these.

We have to focus on managing for the primary materials like rare metals in the long term, considering the energy security. Especially, Korea that core industry is included in high-tech sector like IT must set aside rare metals to improve their competition. In the same context, despite of the biggest producer of rare metals, China is developing Africa mine to obtain necessary resources and United States is supporting the subsidy for the important resources they officially set. EU is also making efforts to reserve the resources together within EU.

The effort for resource securement through development is important, but resource reserve through storage also needs to counteract promptly to the emergency situations. In resource reserve, the effective reserve management is more necessary than just simple reserve due to the large uncertainty around the price.

The purpose of this paper is to summarize the present reserve condition about rare metals and to analyse the efficient reserve management strategy of these. For empirical analysis, the datastream daily price about chrome, molybdenum, selenium and dysprosium(the rare-earth element) was used.

This paper has contributions in two aspects. The first is that this research has leading position of economic analysis for rare metals reserve management in Korea. The second is to calculte the first hitting time which means the used time that present price goes to critical price, in addition to deriving the critical price informs optimal timing to release the reserved stocks.

This paper follows next process. In chaper 2, literature reviews and concept about reserve resource management are introduced, in sequence, in chapter 3, the analysis model using the real option is suggested. In chapter 4, the empirical analysis using the daily price for four kinds of rare metals- chrome, molybdenum, selenium and dysprosium(the rare-earth elements)- is performed and in chapter 5, the summary and conclusion are suggested.

2. Literature review and introduction for reserve concept

① Literature review

There are several literature reviews for reserve resource management. First, Williams et al.(1982) dealt with public and private role in the petroleum management. In this research, the market behavior under the 6 different reserve strategies was considered. Among these 6 ways, 4 cases are dealing with tariff optimized for non-stochastic undistorted cases and these can be again divided in two cases like no price ceiling and price ceiling. Fifth way is the price ceiling and no tariff case and last way is the tariff fully optimized and price ceiling case. Under these 6 ways, the author analysed market price and consumption etc. This paper shows that public storage can really reduce the inverse effect of price ceiling. Proper public storage depends on the tax policy and other policy constraint, in addition to the response in private sector to expected current and future public behaviors.

Oren et al.(1986) studied the optimal SPR(Strategy Petroleum Reserve) and this research is dealing with the optimal size, reserve and release of SPR. Here, optimal policy is decided as minimizing the unstable cost caused by the uncertainty of supply. Petroleum market can be modelled by the elastic demand curve and Markov process of continuous time. The least unstable cost includes consumer loss, reserve cost by the price variations and capital value for the reserve. Moreover the author suggested the basic case result and sensitivity analysis compared to the previous result.

Reznicek et al.(1991) studied the stochastic model about water reserivoir management. The characteristic of hydrologic parameter about water reserivoir management has the uncertainty and in the mathematical programming model of this paper, the uncertainty was considered directly and indirectly by applying constrained opportunity type of formulation or stochastic programming technology.

Howitt et al.(2002) developed the calculation method about the stochastic dynamic programming in resource management. Risk aversion, discount rate and every periodic substitute preference of decision maker are derived from the data used past decisions in the process of minimizing the mean squared error. This paper applied the previous stated structure to the management of water resource having uncertain components. The stochastic dynamic programming which calculated like this recalculated similiarly real historical storage and release. As showing that the sensitivity of optimal decision about the risk aversion and every periodic preference of decision maker, which shows that this model has much lower average expected error than the model deriving risk-neutral expected present value.

Fackler et al.(2002) researched for the optimal storage by crop producer. In this paper, when post-harvest marketing ways are constrained as prohibiting speculative purchases, the sales for storage are irreversible decisions and the dynamic marketing problem becomes similar to the optimal exercise for a financial option. Therefore, the optimal marketing way is purchasing when the price is low and then, is selling when the price is high. In this paper, a method for estimation of cut-off price function which express the boundary between low price and high price was developed and this was applied to soybean price in Illinois. This decision explained by the profit can be obtained through storage.

Zhang et al.(2009) studied stochastic dynamic programming for determining China's optimal strategic petroleum. In this paper, to evaluate the optimal SPR policy, the stocahstic dynamic programming based on potential total cost function that occurs when SPR is reserved was developed. The empirical results show the optimal reserve quantities for some specific cases and optimal reserve-release stategies. Considering these results comprehensively, China's optimal strategy reserve size was calculated as 320 million barrels. This is approximately 90 days in the view of net imported oil amount in 2006. If China follows this plan, national goal for this reserve sould be achieved in 2017, 3 years earlier than orinial goal.

Hughes(2009) studied management of irrigation water storage. In Austrailia, traditionally, given the storage amount, state governments centrally managed for distributing water resource. But, there are a lot of factors which may prevent the centralized approach from achieving allocation of water. This paper shows quantitative analysis that considers two decentralized approaches like carry-over rights and capacity sharing to storage management, involving the application of stochastic dynamic programming model.

Chaton et al.(2009) performed the reaserch about reserve and stability of supply in the midterm. This paper dealt with the role of private storage in a market for the commodity like gas exposing to the irreversible disruption. The analysis considered seasonality for the demand and ignored depletable characteristic. Besides, in a competitive equilibrium including resonable expectation, price and stock were characterized. The result showed robustness of the alternative in the case that the disruption periods or the expected risk continue for finite time. This paper also showed that despite government intervention is necessary, too serious intervention may distort the storage structure.

② Concept of resource reserve

The securement methods for the finite resources that demand exists consist of two aproaches by development or reserve. If market supplies of the resource are stable and exogeneous uncertainty is low, we do not need to care about the reserve in aspect of resource management.

But energy and resource have generally components as finite capacity, regional disparity and cartelization for the production in aspect of supply, along with population growth and industry development in aspect of demand. Previously stated components in demand of energy and resource make their necessity enlarged, according to this, these have the unbalanced factors in demand and supply.

Each country adopts reserve policy for the primary resources have uncertainty and do not exist in each country. Due to these efforts of each country, the competition for resource securement is being more intensive.

In the view of reserve agents, resource reserve can be divided into government reserve and private reserve.

Government reserve means that government stores the resources which is surely necessary in national economy or depend on imports from overseas country. Private reserve means that private firm shortly stores the resources which have high price volatility and need demand and supply stablization. But private reserve has limit in public aspect, because this has to consider the profit of firm.
In the view of reserve purpose, resource reserve can be divided into strategic reserve(duty reserve), buffer stock and operation stock. Strategic reserve means that government or private firm holds reserves for supply trouble to prevent national crisis. Buffer stock means that government or private firm holds the stock to manage business. Operation stock means that private firm holds stock to sustain the production when unbalance of demand and supply in the short term happens.

3. Analytical model

In this chapter, the basic model of real option is used to calculate optimal price which happens release of the reserved stock according to price disruption of rare metals. The demand of rare metals exists only when the industry requires those, due to their characteristics of derived demand. Therefore, in this case, the issue to solve can be summarized to minimizing the negative using cost, given constant demand, *D(t)*.
This paper studies the derivation of optimal price, $P*$ which makes *V(P)* - represent the expected value for *P(t)* - maxmized through dynamic programming, exogeneously given the discount rate, ρ. Here, $P*$ means the timing optimally releases government reserve.
If excess demand which is generated due to shortage of supply when the price suddenly uprises is *E(t),* assume that government has duty to release the quantities corresponding to excess demand. In this case, when government releases *E(t)* from the reserve, the dicount rate of released resource is defined as λ.
P(t) means the price of rare metals which follows GBM(geometric Brownian motion) and this is satisfied with below equation (1). After equation (1), when the equations are expressed, time variable, t will be ommited to simplify the expression.

$$d\,P(t) = \alpha\,P(t)dt + \sigma\,P(t)dw \tag{1}$$

In equation (1), α is growth rate of price, σ is instantaneous volatility of price, dt is increment of time and $W(t)$ follows wiener process. Also, the expected average and variance of wiener process that is continuous stochastic process are defined as $Exp(dw) = 0$ and $Var(dw) = dt$.
Given every periodic demand, $D(t)$, the negative expected value for purchasing reources of a firm before the release of government reserve is defined as $V(P)$. When social discount rate is given as ρ and depletable characteristic of rare metals is not considered, $V(P)$ can be expressed as below equation (2).

$$V(P) = -Exp\int_0^\infty PDe^{-\rho t}\,dt \tag{2}$$

$V(P)$ expressed in equation (2) can be represented as bellman equation of below equation (3) using stochastic dynamic optimization.

$$\rho V(P) = \left\{-PD + \frac{1}{dt}Exp\big(dV(P)\big)\right\} \tag{3}$$

By Ito's Lemma, after $dV(P) = V_P(P)dP + \frac{1}{2}\,V_{PP}(P)(dP)^2$ is calculated, this equation is inserted into equation (3) and then we can insert equation (1) into dP of $V(P)$. After this, using these characteristics, $dt^2 \to 0$, $dt\,dw \to 0$, $Exp(dw) = 0$ and $Var(dw) = dt$, HJB(Hamiltonian-Jacobi-Bellman equation) can be calculated as below equation (4).

$$\rho V(P) = -PD + \alpha P\,V_P(P) + \frac{1}{2}\sigma^2 P^2\,V_{PP}(P) \tag{4}$$

The derived equation (4) means $V_P(P) = dV(P)/dP$ and $V_{PP}(P) = d^2V(P)/dP^2$. Equation (4) is second order differential equation for P which consists of sum of homogeneous and non-homogeneous solution.
Above all, when the initial value of $P(t)$ is defined as P, the solution of non-homogeneous part is equivalent to the expected present value for $P(t)$ and also can be expressed as $Exp(P(t)) = P\,e^{\alpha t}$ using previously

stated conditions. Therefore, by applying this to equation (2), equation (5) is calculated as below. This equation means that non-homogeneous solution of $V(P)$ is the discounted value as much as $(\rho - \alpha)$ for the negative purchasing cost of rare metals in the view of a firm before release of the reserve. Also, for $V(P)$ to have finite value, $(\rho - \alpha) > 0$ should be satisfied.

$$V(P) = -\frac{PD}{(\rho-\alpha)} \tag{5}$$

Next, the solution of homogeneous part means the option value for release of the reserve and has the formation of $V(P) = AP^{\beta}$. If we inserts this result into equation (4), this equation can be arranged as quadratic equation for β characteristic roots of $\rho A(P)^{\beta} = \alpha P\beta A(P)^{\beta-1} + \frac{1}{2}\sigma^2 P^2 \beta(\beta-1)A(P)^{\beta-2}$. In this equation, characteristic roots, β is calculated as below.

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right) + \frac{2\rho}{\sigma^2}}$$

Through above equation, β_1 is found as the positive characteristic satisfying with $\beta_1 > 1$ and β_2 is found as the negative characteristic satisfying with $\beta_2 < 0$. In this case, we can find that the form of homogeneous part solution is $A_1P^{\beta_1} + A_2P^{\beta_2}$. Therefore, the general solution comprises both of homogeneous and non-homogeneous part.

In the economic view for general soultion, $lim_{P\to 0} AP^{\beta} = 0$, in other words, when resource price, P approaches to 0, the option value for release of the reserve converges to 0**.** So, we should accept $A_2 = 0$. When $V(P)$ before the release is defined as V_0, this can be represented like equation (6).

$$V_0 = -\frac{PD}{(\rho-\alpha)} + A_1P^{\beta_1} \tag{6}$$

Next, consider that government replenish the excess demand, $E(t)$ which is generated due to shortage of supply, when price of rare metals sharply increases, from the reserve. In this case, if the discount rate of released resources is λ, $V(P)$ after the release can be defined as V_1 expressed like equation (7).

$$V_1 = -\frac{PD}{(\rho-\alpha)} + \frac{\lambda PE}{(\rho-\alpha)} \tag{7}$$

V_1, value function after the release is equivalently developed as V_0, value function before the release. Merely, the option value does not exist due to release of the reserve and also, according to releasing the reserve which reflects discount rate, $\frac{\lambda PE}{(\rho-\alpha)}$, the benefit present value for cost reduction is added to this value. Release for the reserve will be performed regardless of time if price of rare metals, $P(t)$ exceeds optimal price, $P*$.

Therefore, we can calaulate $P*$, applying value-matching condition and smooth-pasting condition to previously derived V_0, V_1, and this can be arranged like equation (8), (9).

$$-\frac{PD}{(\rho-\alpha)} + A_1P^{\beta_1} = -\frac{PD}{(\rho-\alpha)} + \frac{\lambda PE}{(\rho-\alpha)} - K \tag{8}$$

$$-\frac{D}{(\rho-\alpha)} + A_1\beta_1P^{\beta_1-1} = -\frac{D}{(\rho-\alpha)} + \frac{\lambda E}{(\rho-\alpha)} \tag{9}$$

K means the market price according to release size, therefore K has to be subtracted from equation (8). After this equation is calculated, P* is derived as equation (10).

$$P^* = \frac{\beta}{(\beta-1)} K \frac{(\rho-\alpha)}{\lambda E} \tag{10}$$

If the initial value of $P(t)$ is P, the distribution of time, τ to arrive at $P*$ is given by the inverse Gaussian function and this is represented as the following.

$$F(\tau\,;\,\mathrm{P}) = \frac{P^{*}-P}{\sqrt{2\pi\sigma^{2}\tau^{3}}}Exp\left[-\frac{(P^{*}-P-\alpha\tau)^{2}}{2\sigma^{2}\tau}\right] \quad (11)$$

Through this equation, if caluculating the first hitting time to $P*$, this can be expressed as the following.

$$Exp(\tau^{*}) = \begin{cases} \frac{1}{\alpha-0.5\sigma^{2}}\ln\left(\frac{\mathrm{P}^{*}}{\mathrm{P}}\right) & if\ \alpha > 0.5\,\sigma^{2} \\ \infty & if\ \alpha \leq 0.5\,\sigma^{2} \end{cases}$$

Next, by using previously derived model and applying the datastream price of selenium, dysprosium, molybdenum and ferro-chrome to this model, the empirical analysis for price trend and reserve management will be proceeded.

4. Empirical analysis

① Analysis of price trend

The supply of selenium produced as by-product is directly affected by the supply of copper and the price of selenium is inverse proportion to the supply of copper production. For example, the price of selenium produced as refining of copper decreases if the production of copper increases, according to the unbalance of selenium production for demand and supply in 2003 and 2004, the price of selenium rose. Also, at free market, selenium is hard to be bought because the supply of seleium has been contracted in the long term. By this, the price of spot market remarkably increased from 2002 to 2005.

In 2010, national selenium consumptions of United States in glass and copy machine sector continuously reduced and using as the substitute of lead slightly increased. Besides, the demand as vitamin supplement drug and as solar battery material is increasing.

By these factors, the price of selenium was about $80/Kg in 2005 and about $40/KG between 2006 and 2007. After that, the price continuously increased to about $120/Kg in september, 2011.

Molybdenum market is primarily formed by the brokers of Europe, North America and Japan as spot and foward trading condition. The copper mine firms are producing molybdenum has been seized as much as 80% of total production and molybdenum supply is depending on copper production.

The price of molybdenum oxide in 2004 increased 3 times more than 2003, the one in 2005 increased twice more than 2004. In 2008, the price of molybdenum oxide was mostly stable from about $30/Kg to about $34.5/Kg, but the sales was overweighed by the reduced production of stainless factories. The first quater price of molybdenum in 2009 was downward due to serious recession, but the price started to increase from the third quater and the price in first half year of 2010 was upward due to the optimistic view for the demand of molybdenum.

Figure 1. Log-difference for selenium price (2005.12.16.-2011.9.23)

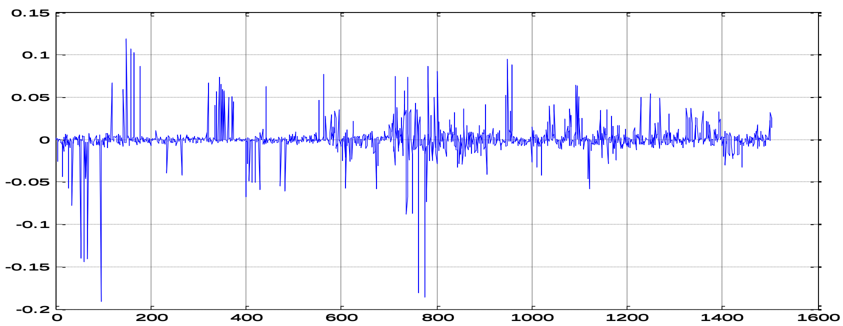
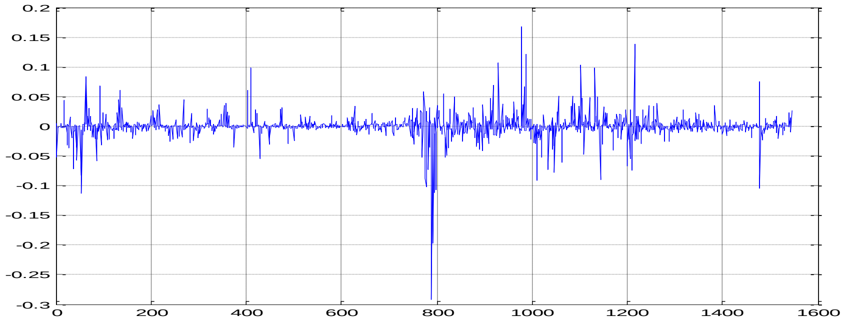
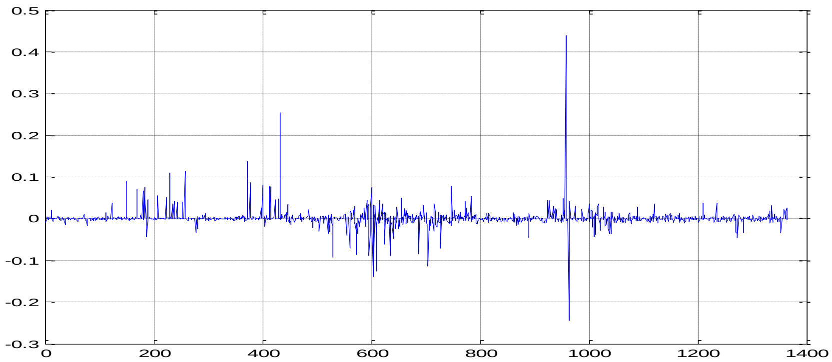


Figure 2. Log-difference for molybdenum oxide price (2005.10.21-2011.9.23)



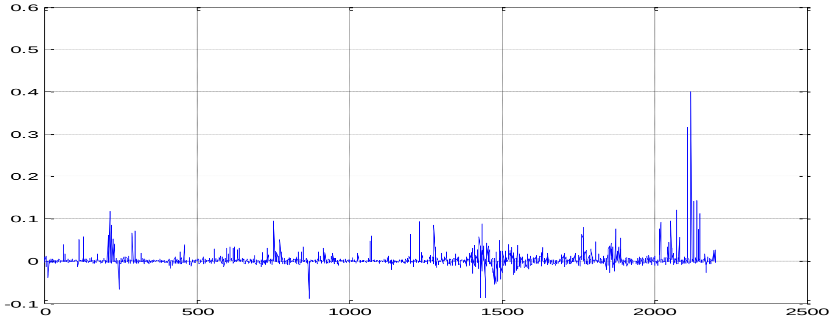
In the case of ferro-chrome, the factories show the trend of location from stianless production country to production country for chrome ore. This production needs much electricity and the required power for production of 1 ton ferro-chrome is 2900~4100Kwh. The efficiency is different according to the quality of ore and operational environment. Therefore, the location of factory reflects on the cost equilibrium between material ore and power supply.
The trend of ferro-chrome price represents high volatility. In the mid term of 2006, the price started from about $1.4/Kg and sharply rose to about $6/Kg in the mid term. After then, the price again was downward and now, this is transacting about $2.5/Kg.

Figure 3. Log-difference for ferro-chrome price (2006.7.1.-2011.9.23)



The price decision of rare earth elements is affected from China in aspect of supply and United States or Japan in aspect of demand. China is in charge of above 95% of total rare earth elements production and recently through merging the small firms for the rare earth elements production, China government is performing resource-nationalism policy. In 2008, the price was downward for a while, but the one was again upward.
In this case, dysprosium, a kind of rare earth elements was transacted about $28/Kg, but in according to resource-nationalism policy and industrial demand, the price has contiuously increased and now, the transacted price is surprisingly about $3000/Kg.

Figure 4. Log-difference for dysprosium price (2003.4 – 2011.9)



② Analysis for reserve management

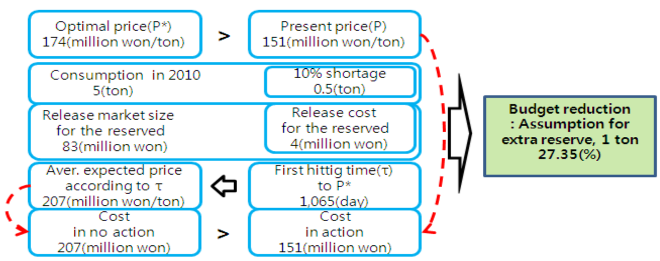
The results that the drift and volatility rate are estimated, through applying log-normal distribution to time series data for selenium price, is $\alpha = 0.0003$ and $\sigma = 0.0018$. The optimal price for release of the reserve by government, $P*$ was calculated as 174 million won/ton[3]. Through this result, we can analyse that present price is not up to the optimal value $P*$ because this optimal price is larger than 151 million won/ton, the price in 23, september, 2011.

Annual selenium consumption in 2010, as standard, is 5ton and let's assume that the shortage of supply happens as much as 10% of total annual selenium consumption in 2010. Then, government has to release the reserve, 0.5ton. In this case, release market size and release cost are each expressed as $(1-\lambda) \times P^* \times E$ and $\lambda \times P^* \times E$. Through each calculation equation, release market size and release cost are derived as 83 million won and 4 million won. Also, the expected first hitting time to optimal price, $P*$ is calculated as $Exp(\tau) = 1{,}065$ days.

When this hitting time is set as standard, the average expected price applied expected value of GBM is calculated as $E(P) = P_0 \exp(\alpha\tau) = 207$ million won/ton. Along with this, when above result is set as standard, the reserve budget is calculated as (expected price × release cost). After this, when assuming that extra reserve 1ton is necessary, "the cost of no action(A)" that generates for extra reserve 1ton when the price becomes p* is calculated as 207 million won by (207 million won/ton × 1 ton). Also, in the case of early reserve that begins to reserve now, "the cost for early reserve(B)" is calculated as 151 million won by (151 million won/ton × 1ton). In this case, the effect for budget reduction by early reserve is analysed as (A-B)/A = 27.35%.

The results that the drift and volatility rate are estimated, through applying log-normal distribution to time series data for the molybdenum oxide price, is $\alpha = 0.0006$ and $\sigma = 0.0196$. The optimal price for release of the reserve by government, $P*$ was calculated as 44 million won/ton. Through this result, we can analyse that present price is not up to the optimal value $P*$ because this optimal price is larger than 38 million won/ton, the price in 23, september, 2011. In this case, the radical change of structure for this time seiries price is obtained, therefore, following research using the extreme value is considered to be necessary.

Figure 5. Selenium reserve management analysis

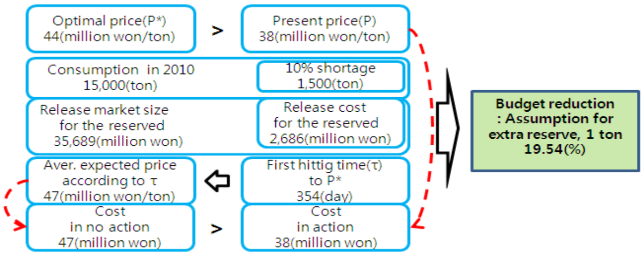


[3] 1 $ = 1,156.90 won(23, september, 2011)

Annual molybdenum oxide consumption in 2010, as standard, is 15,000ton and let's assume that the shortage of supply happens as much as 10% of total annual molybdenum oxide consumption in 2010. Then, government has to release the reserve, 1,500ton. In this case, release market size and release cost are each expressed same as the ones about selenium. Through this method, release market size and release cost are obtained as 35,689 million won and 2,686 million won. Also, the expected first hitting time to optimal price, $P*$ is calculated as $Exp(\tau) = 354$ days.

When this hitting time is set as standard, the average expected price applied expected value of GBM is calculated as $E(P) = P_0 \exp(\alpha\tau) = 47$ million won/ton. Along with this, when above result is set as standard, the reserve budget is calculated as (expected price × release cost). After this, when assuming that extra reserve 1ton is necessary, "the cost of no action(A)" that generates for extra reserve 1ton when the price becomes p* is calculated as 47 million won by (47 million won/ton × 1 ton). Also, in the case of early reserve that begins to reserve now, "the cost for early reserve(B)" is calculated as 38 million won by (38 million won/ton × 1ton). In this case, the effect for budget reduction by early reserve is analysed as (A-B)/A = 19.54%.

Figure 6. Molybdenum oxide reserve management analysis

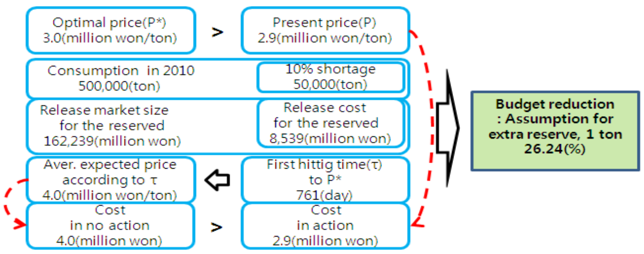


The results that the drift and volatility rate is estimated, through applying log-normal distribution to time series data for ferro-chrome price, is $\alpha = 0.0004$ and $\sigma = 0.020$. The optimal price for release of the reserve by government, $P*$ was calculated as 3.0 million won/ton. Through this result, we can analyse that present price is not up to the optimal value $P*$ because this optimal price is larger than 2.9 million won/ton, the price in 23, september, 2011.

Annual ferro-chrome consumption in 2010, as standard, is 500,000ton and let's assume that the shortage of supply happens as much as 10% of total annual ferro-chrome consumption in 2010. Then, government has to release the reserve, 50,000ton. In this case, release market size and release cost are each expressed same as above method. Through this method, release market size and release cost are obtained as 162,239 million won and 8,539 million won. Also, the expected first hitting time to optimal price, $P*$ is calculated as $Exp(\tau) = 761$ days.

When this hitting time is set as standard, the average expected price applied expected value of GBM is calculated as $E(P) = P_0 \exp(\alpha\tau) = 4.0$ million won/ton. Along with this, when above result is set as standard, the reserve budget is calculated as (expected price × release cost). After this, when assuming that extra reserve 1ton is necessary, "the cost of no action(A)" that generates for extra reserve 1ton when the price becomes p* is calculated as 4.0 million won by (4.0 million won/ton × 1 ton). Also, in the case of early reserve that begins to reserve now, "the cost for early reserve(B)" is calculated as 2.9 million won by (2.9 million won/ton × 1ton). In this case, the effect for budget reduction by early reserve is analysed as (A-B)/A = 26.24%.

Figure 7. Ferro-chrome reserve management analysis



The results that the drift and volatility rate is estimated, through applying log-normal distribution to time series data for dysprosium price, is $\alpha = 0.006$ and $\sigma = 0.028$. The optimal price for release of the reserve by government, $P*$ was calculated as 4,061 million won/ton. Through this result, we can analyse that present price is not up to the optimal value $P*$ because this optimal price is larger than 3,524 million won/ton, the price in 23, september, 2011.

Annual dysprosium consumption in 2010, as standard, is 41ton and let's assume that the shortage of supply happens as much as 10% of total annual dysprosium consumption in 2010. Then, government has to release the reserve, 4.1ton. In this case, release market size and release cost are each expressed same as above method. Through this method, release market size and release cost are obtained as 15,850 million won and 834 million won. Also, the expected first hitting time to optimal price, $P*$ is calculated as $Exp(\tau) = 25$ days. This hitting time may result from recent uprising of the price, so, we need to consider drop of the price according to the quantity adjustment.

When this hitting time is set as standard, the average expected price applied expected value of GBM is calculated as $E(P) = P_0 \exp(\alpha\tau) = 4{,}101$ million won/ton. Along with this, when above result is set as standard, the reserve budget is calculated as (expected price × release cost). After this, when assuming that extra reserve 1ton is necessary, "the cost of no action(A)" that generates for extra reserve 1ton when the price becomes p* is calculated as 4,101 million won by (4,101 million won/ton × 1 ton). Also, in the case of early reserve that begins to reserve now, "the cost for early reserve(B)" is calculated as 3,524 million won by (3,524 million won/ton × 1ton). In this case, the effect for budget reduction by early reserve is analysed as (A-B)/A = 14.08%. In the case of dysprosium, the price has enough possibility to be downward and as a result, the incentive for reserve is weaker than other analysed things.

Figure 8. Dysprosium reserve management analysis

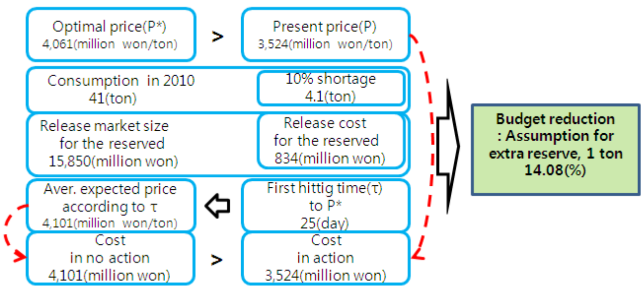


Figure 9. Sensitivity analysis for optimal price according to the social discount rate and price discount rate.

5. Summary and Conclusion

Currently, increasing oil price and commodity price are observed by instability of international situations and resource nationalism. At the same time, the instablility in aspect of demand and supply of the resources is showing upward trend. By this, now the importance of resource reserve becomes larger than any past period.
Besides, the competition for resource securement among countries have little resources is being serious and the offensive polilcy like export constraint of China for rare earth elements is being explicit. By these changes, the national competition for resource securement is expected to be tougher than past.
In the global market, Korea that is sharply developing in high-tech industry like IT and Automobile has little natural resources, so, the materials for production are totally imported from abroad. In other words, if supply shock is happened, that will fast affect to Korea. Therefore, policy makers have to consider the reserve problem in the long term, including energy security for stable supply.
In this paper, the optimal reserve management of rare metals which industrial demands exist is analysed. Real option analysis was used to find the optimal release time of the reserve and the empirical analysis was performed for four kinds of rare metals, selenium, molybdenum, chrome and dysprosium.
First, we can find that the present price of analysed four kinds of rare metals did not reach the optimal price, $P*$ through the result.
The first hitting time of each rare metals from present price to the optimal price was found as 1065, 761, 354 and 25 days for the selenium, ferro-chrome, molybdenum oxide and dysprosium. In this case, the hitting time of dysprosium is remarkably short due to the recent price uprising by resource nationalism policy and industrial demand increasing, but if the extra productions of America and India are reoperated, the price would be likely to decrease.
The result for early reserve was calculated as selenium, (27.35%), ferro-chrome, (26.24%), molybdenum, (19,54%) and dysprosium, (14.08%). In this case, the result shows that the incentive of dysprosium is lower than other things.
Although rare metals are affected by volatility of price and export policy, considerable approaches about the reserve of these in the long term are necessary due to the important position in energy security. The result of this paper shows that the analysed four kinds of rare metals need the early reserve.
In later policy, along with the securement of abroad resources, the urban mine has to be actively utilized. The urban mine means the business which recycles useful materials extracted in the dicarded electronic items and industrial waste. In Korea, we need to care about the potential capacity and management because there are a lot of high-tech items including useful rare metals. For instance, Japan consists of the largest urban mine and obtains several rare metals from this. The capacity is not small compared to other countries and probably is known that the size is up to about 40,000 billion yen.(Jung, 2011). Except for analysed four rare metals, we will research later about the optimal reserve management for several primary resources and urban mine.

Reference

Hojeong Park and Eunchun Park, 2010. Real Option Analysis of the North-East Oil Hub Project in Korea, Applied economics, Vol 12, No 1

Hosung Jung, 2011, Evaluation for the potential value of rare metals in Korea urban mine, SERI Issue paper

Hanshin Choi, Yonghwan Kim and Youlgwun Ji, 2010, Overview on Rare Metal Industries and Their Recycling, KIC News, Volume 13, No. 1, 2010

Ministry of Knowledge Economy, The demand and supply of mineral products and price trend, 2011

Ministry of Knowledge Economy, The investment for mineral products, 2011

Public Procurement Service, Stockpile Business Manual

KOMIS, http://www.kores.net/

Corinne Chaton, Anna Creti, Bertrand Villeneuve, 2009, Storage and Security of Supply in the Medium Run, Resource and Energy Economics, Vol. 31, No. 1(Jan. 2009), pp 24-38

Jeffrey C. Williams and Brian Davern Wright, 1982, The role of public private storage in managing oil import disruptions, The Bell Journal of Economics, Vol. 13, No. 2(Autumn. 1982), pp 341-353

K. Reznicek and T.C.E Chen, 1991, Stochastic modelling of reservoir operations, Euuropen Journal of Operational Research 50, 235-248

Neal Hughes, 2009, Management of irragation water storage: carryover rights and capacity sharing, Australian Agriculture and Resource Economics Society

Paul L. Fackler and Michael J. Livingston, 2002, Optimal Storage by Crop Producer, American Journal of Agricultural Economics, Vol. 84, No. 3(Aug, 2002), pp 645-659

P.Wilmott, Paul Wilmott on Quantitative Finance, Wiley. NY, 2000

Richard E.Howitt, Arnaud Reynaud, Siwa Msangi, and Keith C. Knapp, 2002, Calibrated Stochastic Dynamic Models For Resource Management, World Congress of Environmental and Resource Economics

Shmuel S.Oren and Shao Hong Wan, 1986, Optimal strategic pteroleum reserve policies-a steady state analysis, Management Science Vol. 32, No1

Xiao-Bing Zhang, Ying Fan, Yi-Ming Wei, 2009, A model based on stochstic dynamic programming for determining China’s optimal strategic petroleum reserve policy, Energy Policy Vol. 37, pp4397-4406