Uncertainty and Investment Option Games in Emission Permit Market

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Abstract

This paper analyzes the investment effects of tradable permit programs when the abatement cost is uncertain and the pollution abatement investment is competitively determined within game-theoretic framework. As well known in the real option literature, uncertainty provides negative impacts on environmental investment. A dynamic stochastic model is employed to consider the irreversibility of investment and cost uncertainty. The condition for competitive firm's investment threshold is derived and the effect of such competitiveness on environmental investment is examined. The result shows that the effect of investment on abatement cost and allowance price acts in an opposite way through firms' preemptive incentive in the permit market.

1 Introduction

This paper introduces an oligopolistic permit market in which the permit price is affected by the aggregate stock of abatement capital. This provides an environment for an individual firm to behave as if it can keep perpetually its monopolistic opportunity for the investment no matter how the other firms react. Obviously, such monopolistic opportunity setting makes a firm's investment option valuation independent of the other firms' investment strategies.

A real option model is developed to allows for the investment behavior of each firm to depend on other firms through permit market participation. In a strategic environment, the value of investment is endogenously determined and the optimal investment threshold cannot be derived in isolation, but must be evaluated within a game-theoretic

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framework. This paper aims to provide a tractable solution for deriving the equilibrium investment strategies of firms being regulated by a TPP. A symmetric Cournot-Nash equilibrium that is identified conditional on its competitors is determined when each firm simultaneously decides its equilibrium investment strategy. It is shown that the results derived in a single firm's investment model remains valid as in Park (2008). More interestingly the model analyzes the effect of competing firms on each firm's investment decision rule.

There are several ways to construct a real option model in which each firm considers its option value contingent on other firms' strategies. I adopt the "myopic" solution approach of Leahy (1993). Under this approach, each firm is farsighted in the sense that it evaluates present values. On the contrary, it is shortsighted in the sense that the process to calculate present value is valid only so long as no other firms invest. Using this strategy, Leahy (1993) developed a model in which each firm competes with other firms to maximize its profit in oligopolistic output market when profit uncertainty prevails.

Extending Leahy's (1993) model, Grenadier (2002) recently presented option exercise game model to analyze the impact of competition on investment timing when there exists an opportunity other firms preempt investment opportunities. He showed that increasing competition between firms erodes valuable option to wait. Both Leahy (1993) and Grenadier (2002) analyzed profit uncertainty and oligopolistic output market.

This paper focuses on analyzing the investment effect of competitiveness under a TPP. The analysis follows Grenadier's (2002) extension of Leahy's model since it has the advantage that is more general and tractable, and it can be used to analyze the effect of increasing competitiveness on investment.

2 Model

Consider that there are n identical firms in a permit market. The degree of competitivenss is denoted by the number of firms, n. Initially, those n firms differ in their abatement capital k, but are identical otherwise. Two types of interaction between firms can be envisioned. First, increases in abatement capital may affect the permit price by reducing permit demand. We call this the 'price effect' of abatement capital. Second, due to technological spillover, a firm's abatement cost may be affected by the aggregate stock of abatement capital. We call this second effect the 'cost effect' of capital.

Under TPP, each firm with the baseline emission rate $e(t)$ receives emission allowances $\bar{e}(t) < e(t)$ from a regulatory agency. Each firm abates emissions at rate $a(t) \leq e(t)$. Permits are purchased at rate $q(t)$. If a firm reduces its emissions below $\bar{e}(t)$, excessive permits can either be sold at $q(t) < 0$. The market for emission permits is assumed to be competitive so that firms take the permit price, $p(t)$, as given. Emissions, abatement, and permit transactions satisfy the accounting identity:

$$q(t) = e(t) - \bar{e}(t) - a(t). \tag{1}$$

In what follows, time t is suppressed for notational convenience unless it is needed for clarity.

Emission abatement costs depend on installed abatement capital, $k(t)$, the instantaneous rate of abatement, $a(t)$, and a parameter, θ, that represents industry-wide cost uncertainty common to all firms. This paper will focus on a symmetric Nash equilibrium in which $k_i^*(t) = k_j^*(t)$ for all $i, j = 1, ..., n$. Let the abatement cost function be denoted as

$$C(\theta, k_i) = \theta c(k_i + K_{-i}) a^2 \tag{2}$$

where $K_{-i} = \sum_{j=1, j \neq i}^{n} k_j$ as a function of aggregate capital to take into account 'cost effect'. The term $c(k)$ captures the effect of installed capital on abatement costs. It is assumed that $c'(k) < 0$. The implication is that a firm can reduce its future abatement costs by investing in more efficient abatement capital. At each instant the firm must decide whether to undertake investment and expand captial from k to $k + dk$, or maintain its current level of k without any adjustment. The unit cost of capital is w. Investment is considered irreversible so that $dk > 0$, and for simplicity, there is no depreciation.

Current abatement cost is known but there is uncertainty over future abatement costs. Uncertainty is represented by assuming the cost parameter, θ, follows the geometric Brownian motion stochastic process:

$$d\theta = -\alpha\theta dt + \sigma\theta dz \tag{3}$$

where dz is the increment of a standard Wiener process, uncorrelated over time, with

$E(dz) = 0$, $Var(dz) = dt$ and $\theta(0) = \theta_0 \geq 0$. The drift parameter, $-\alpha$, measures the expected growth rate of the stochastic process. The fact that it is negative implies that firms face uncertainty over cost reducing technical change. The parameter σ represents the volatility rate of the stochastic process and $\sigma > 0$ implies that the variance of future costs increases with the time horizon over which forecasts are being made. The parameter γ is the elasticity of cost with respect to abatement. To simplify the presentation and obtain an explicit analytical solution we assume a quadratic specification where $\gamma = 2$. Total compliance cost is given by abatement cost plus permit purchase cost (or less permit sales revenue). Using (1) and (2) this can be expressed as:

$$C(a, \theta, k) + pq = \theta c(k)a^2 + p(e - \bar{e} - a + b). \tag{4}$$

The decision problem for the firm can be summarized as follows. Given the state, $(\theta(t), k(t))$, the firm chooses a policy for abatement, permit transactions, and investment in abatement capital to minimize the expected discounted stream of costs over time.

The ith firm's total compliance cost is denoted by $\theta c(k_i+K_{-i})a^2+p(k_i+K_{-i})(e-\bar{e}-a)$ given any permit price $p(k_i + K_{-i})$ that is competitively determined as described soon. Note that permit price is a function of the aggregate abatement capital to incorporate 'price effect'. Each firm faces market wide uncertainty (3) regarding technological progress.

Let $V^i(\theta, k_i(t), K_{-i}(t))$ denote the value of firm i, for given other firms' strategies $K_{-i}(t)$. Then the decision problem for firm i is given by:

$$V^i(\theta, k_i, K_{-i}) = \max_{a, \Delta k_i} -E_0 \int_0^{\infty} (\theta c(k_i+K_{-i})a^2+p(k_i+K_{-i})(e-\bar{e}-a))e^{-rt}dt - we^{-r\tau^*}dk_i \tag{5}$$

subject to (3). The substitution of optimal abatement schedule, $a = p(k_i + K_{-i})/2\theta c(k_i+K_{-i})$, into (5) leads to the following constrained HJB equation:

$$rV^i(\theta, k_i, K_{-i}) = \theta^{-1}\pi(k_i + K_{-i}) - p(k_i + K_{-i})(e - \bar{e}) - \alpha\theta V^i_\theta + \frac{1}{2}\sigma^2\theta^2 V^i_{\theta\theta} \tag{6}$$

where $\pi(k_i + K_{-i}) = p(k_i + K_{-i})^2 / 4c(k_i + K_{-i})$. Each firm holds a sequence of investment opportunities that is analogous to a call option. At any point in time, each firm can invest to increase its capital by an infinitesimal increment dk_i at linear adjustment cost w. Since the optimal investment trigger must be determined endogenously, I look at a Nash equilibrium solution in exercise strategies. Each firm chooses its investment process so as to maximize its value, conditional on the assumed stochastic process and exercise strategies of its competitors. Thus the strategies $[k_1^*(t), k_2^*(t), ..., k_n^*(t)]$ constitutes a Nash equilibrium if

$$V^i(\theta, k_i^*(t), K_{-i}^*(t)) = \sup_{\Delta k_i} V^i(\theta, k_i(t), K_{-i}^*(t)) \text{ for all } i. \tag{7}$$

The competitive equilibrium permit price is accordingly determined in a market-clearing level:

$$p^*(t) = p(k_i^*(t) + K_{-i}^*(t)) \text{ such that } 2\theta c \left(k_i^* + K_{-i}^*\right) a = p\left(k_i^* + K_{-i}^*\right) \tag{8}$$

Equation (7) and (8) characterize the competitive equilibrium. Now consider firm i's optimal investment strategy contingent on the investment strategy of its competitors. Suppose that other firms are assumed to incrementally increase capital capacity whenever $\theta(t)$ rises to $\theta_{-i}^*(k_{i,}K_{-i})$. Then, the value function of ith firm becomes dependent of $\theta_{-i}^*(k_i, K_{-i})$.

The first boundary conditions that V^i must satisfy are value-matching and smooth-pasting conditions:

$$\begin{aligned} \frac{\partial V^i}{\partial k_i}(\theta, \theta_i^*, k_i, K_{-i}; \theta_{-i}^*) &= w, & (9) \\ \frac{\partial^2 V^i}{\partial k_i \partial \theta}(\theta, \theta_i^*, k_i, K_{-i}; \theta_{-i}^*) &= 0. & (10) \end{aligned}$$

Additional boundary condition is imposed to incorporate strategic interaction between firms. At θ_{-i}, firm i's competitors exercise, K_{-i} increases by the infinitesimal increment dK_{-i}. The final boundary condition is a value-matching condition that relates V^i to the competitors' investment threshold, $\theta_{-i}(k_i, K_{-i})$. Thus at the moment of competitive exercise, $V^i(\theta, \theta_i^*, k_i, K_{-i}; \theta_{-i}^*) = V^i(\theta, \theta_i^*, k_i, K_{-i} + dK_{-i}; \theta_{-i}^*)$. Dividing by the incremental dK_{-i}, this can be written as

$$\frac{\partial V^i}{\partial K_{-i}}(\theta, \theta_{-i}^*, k_i, K_{-i}; \theta_{-i}^*) = 0. \tag{11}$$

Since three boundary conditions (9), (10) and (11) should be solved simultaneously, the determination of a Nash equilibrium becomes a complex fixed point problem. As

Grenadier (2002) proves, however, the problem can be simplified into a standard real option problem in which only two boundary conditions, (9) and (10), remain. He demonstrates that when an individual firm in the equilibrium investment policy adopts a myopic strategy, the other firms' exercises can be ignored. Given a current level of competitive investment K_{-i}, a myopic firm assumes that K_{-i} will remain fixed forever. More precisely, let $\tilde{V}^i(\theta, k_i, K_{-i})$ and $\tilde{\theta}_i^*$ denote the value of a myopic firm and the resulting investment trigger, respectively. The following proposition states that the myopic investment trigger is idential to the Nash equilibrium investment trigger.

Proposition 1 *The symmetric Nash equilibrium investment strategy is characterized by each firm increasing abatement capital whenever $\theta(t)$ reaches down to the myopic trigger level $\tilde{\theta}_i^*(k_i, K_{-i})$ and it is determined by the following differential equation*

$$r\tilde{V}^i(\theta, k_i, K_{-i}) = \theta^{-1}\pi(k_i + K_{-i}) - p(k_i + K_{-i})(e - \bar{e}) - \alpha\theta\tilde{V}_\theta^i + \frac{1}{2}\sigma^2\theta^2\tilde{V}_{\theta\theta}^i \tag{12}$$

and the boundary conditions

$$\frac{\partial \tilde{V}^i}{\partial k_i}(\tilde{\theta}_i^*, k_i, K_{-i}) = w \tag{13}$$

$$\frac{\partial^2 \tilde{V}^i}{\partial k_i \partial \theta}(\tilde{\theta}_i^*, k_i, K_{-i}) = 0. \tag{14}$$

Lemma 1 V_k^{i*} *is continuous in θ. $\theta_i^*(k_i, K_{-i})$ is a continuous correspondence in k_i and K_{-i}.*

Detail proof is not presented here but it needs to be worthy of noting that V_k^i is continuous in θ since θ exhibits continuity and infinite variation according to the underlying stochastic process. Therefore, the first statement is the result from the Theorem of the Maximum (Stokey and Lucas (1989), Leahy (1993)). Note that this representation is identical to the system for V^i with the exception that competitors' trigger θ_{-i}^* is ignored. Using above proposition, (12) and (14) can be solved to obtain the optimal investment threshold under a permit program. For notational brevity, denote $\varsigma_c = -\tilde{c}'(K)\,K/\tilde{c}(K)$ and $\varsigma_p = -\tilde{p}'(K)\,K/\tilde{p}(K)$ as elasticity of abatement cost and permit price, respectively, with respect to abatement capital.

Let θ^* denote the optimal investment threshold under a TPP. The HJB equation becomes:

$$rV(\theta,k) = \pi(k)\,\theta^{-1} - p(e-\bar{e}) - \alpha\theta V_\theta(\theta,k) + \frac{1}{2}\sigma^2\theta^2 V_{\theta\theta}(\theta,k)\,. \tag{15}$$

It is natural to require that a solution to this equation should satisfy the boundary condition is introduced:

$$\lim_{\theta\to\infty} V(\theta,k) = -p(e-\bar{e})/r \tag{16}$$

so that no abatement occurs when the abatement cost is infinite and the expected present value of compliance consists of only the permit purchase cost. Using the method of undetermined coefficients, a particular solution for the non-homogeneous component of (15) is given as

$$V^p(\theta,k) = \frac{\pi(k)}{\theta\,(r-\alpha-\sigma^2)} - \frac{p(e-\bar{e})}{r}. \tag{17}$$

Superscript p denotes the particular solution. The first term on the right hand side represents the present value of net abatement benefit at currently installed k. The investment option value is obtained by solving the homogeneous part of (15), $rV = -\alpha\theta V_\theta + (1/2)\,\sigma^2\theta^2 V_{\theta\theta}$. The general solution denoted with superscript g is given by

$$V^g(\theta,k) = A_1(k)\theta^{\phi_1} + A_2(k)\theta^{\phi_2} \tag{18}$$

where $A_1(k)$ and $A_2(k)$ are constants to be determined using additional boundary conditions. ϕ_1 and ϕ_2 are the positive and negative roots, respectively, of the characteristic equation $\Omega = 0.5\sigma^2\phi_N(\phi_N-1) - \alpha\phi_N - r$:

$$\phi_1 = \frac{1}{2} + \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2}+\frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{19}$$

$$\phi_2 = \frac{1}{2} + \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2}+\frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < -1. \tag{20}$$

If $A_1(k) \neq 0$, then $\phi_1 > 1$ implies $\lim_{\theta\to\infty} V^g(\theta,k) = \infty$. Since this violates the boundary condition (16) the first option term in (18) is eliminated by setting $A_1(k) = 0$. The intuition is that infinite abatement costs are sufficient to deter any investment in abatement capital. Given $A_1(k) = 0$, we can simplify notation by removing the subscript 2. Combining (17) and (18), the solution for the value function V is given by

$$V(\theta,k) = \frac{\pi(k)}{\theta\,(r-\alpha-\sigma^2)} - \frac{p(e-\bar{e})}{r} + A(k)\theta^{\phi}. \tag{21}$$

The properties of ϕ (i.e., ϕ_2) are summarized as follows:

Remark 1 *(i)* $\phi < -1$, *(ii)* $\partial\phi/\partial\sigma > 0$ *(iii)* $\partial\phi/\partial\alpha > 0$, *and (iv)* $\partial\phi/\partial r < 0$

To determine the optimal investment trigger θ^* and the constant term $A(k)$, the value-matching condition (22) and super-contact condition (23) are required (Dumas, 1991):

$$V_k(\theta^*, k) = w, \tag{22}$$

$$V_{k\theta}(\theta^*, k) = 0. \tag{23}$$

The value-matching condition states that the marginal value of investment is equal to the marginal capital adjustment cost at the optimal threshold of θ^*. The super-contact condition (23) allows a smooth transition from the no-investment regime to the investment regime. By solving two boundary conditions simultaneously, we have

Lemma 2 *The optimal investment threshold that specifies an investment rule, (22) and (23), is*

$$\theta^* = \frac{\pi'(k)}{(r - \alpha - \sigma^2)} \left(\frac{1}{wH}\right) > 0 \tag{24}$$

where $H = \phi/(\phi + 1)$.

Proof. The derivation of θ^* is straightforward from two boundary conditions, (22) and (23). Note that they can be explicitly rewritten as

$$A'(k)\theta^{*\phi} = \frac{-\pi'(k)}{\theta^*(r - \alpha - \sigma^2)} + w \tag{25}$$

$$\phi A'(k)\theta^{*\phi} = \frac{\pi'(k)}{\theta^*(r - \alpha - \sigma^2)} \tag{26}$$

where $\pi'(k) = -c'(k)\, p^2/4c(k)^2$. By substituting $A'(k)\theta^{*\phi}$ of (26) into (25), and then rearranging terms, θ^* is obtained as (24). Since $r - \alpha - \sigma^2 > 0$ and $H > 0$ from the assumption A.1, θ^* is an intuitively valid threshold that is positive. ■

Note that θ^* in (24) is represented with the marginal expected present value of net abatement benefit devided by the capital adjustment cost, w, augmented with the option value multiple, H. We shall discuss more about H shortly. The LHS in (25) is the marginal option value for an investment and the RHS in (25) represents the marginal change of the expected present value of abatement plus the capital adjustment cost. A waiting value at θ is measured by $-A'(k)\theta^{\phi} - \left[\frac{\pi'(k)}{\theta(r-\alpha-\sigma^2)} - w\right]$. Until θ reaches θ^*, a positive waiting value prevails but at $\theta \leq \theta^*$, no more waiting value exists. Therefore, the value-matching condition implies that the waiting value becomes zero at θ^* and it is optimal for the firm to adjust its capital immediately.

The option value multiple H captures hysteresis effect in a non-bankable TPP, measuring degree of reluctance to undertake an investment. The larger value of H indicates greater reluctance to adjust capital since it lowers the investment threshold. From Remark 1, following properties of H are immediate:

Lemma 3 *(i) $\partial H/\partial\sigma > 0$, (ii) $\partial H/\partial\alpha > 0$, and (iii) $\partial H/\partial r < 0$.*

As consistent with conventional real option result, larger uncertainty creates larger hysteresis: $\partial H/\partial\sigma > 0$. The property of (ii) of Lemma 2 is obtained because, when it is expected that technological progress will reduce the abatement cost rapidly due to larger value of α, the attraction for an immediate investment may decrease and investment hysteresis arises. In this context, α can be associated with the opportunity cost of exercising the investment option (Dixit and Pindyck, 1994; Trigeorgis, 1996). As we shall see in the next section, this property provides a critical distinction between investment thresholds under a bankable TPP and a non-bankable TPP.

Now, combined with Lemma 2, Lemma 1 develops following comparative statics on θ^*:

Lemma 4 *(i) $\partial\theta^*/\partial w < 0$, (ii) $\partial\theta^*/\partial p > 0$, but (iii) the signs of $\partial\theta^*/\partial\sigma$, $\partial\theta^*/\partial\alpha$, and $\partial\theta^*/\partial r$ are indeterminate.*

An increase in irreversible investment cost makes a firm reluctant to expand its capital capacity and consequently reduces the investment threshold. On the other hand, an increase in permit price raises the investment incentive. However, the effects of α, σ and r are not determinate because these parameters have opposite effects on the option value multiple and the marginal expected present value of abatement. For example, suppose σ is increased. Then greater uncertainty yields larger H which reduces the level of

θ^*. However, greater uncertainty results in a lager marginal expected present value of abatement, increasing the level of θ^*. Consequently, the total effect of uncertainty on θ^* comes to be determined depending on the value of underlying parameters.

Proposition 2 *If $\frac{1}{2}\varsigma_c > \varsigma_p$, the oligopolistic firm's investment threshold $\tilde{\theta}^*$ exists and it is defined by*

$$\tilde{\theta}^* = \frac{\frac{\widetilde{p}(K)^2}{2\widetilde{c}(K)}\left(\frac{1}{2}\varsigma_c - \varsigma_p\right)n}{\left(n\widetilde{M}'(k) + w\right)H^v(r-\alpha-\sigma^2)K} \tag{27}$$

where $M(k_i + K_{-i}) = p(k_i + K_{-i})(e - \bar{e})$, $H^v = \beta^v / (\beta^v + 1)$ and β^v is given by (20).

Proof. For notational simplicity, let

$$\begin{aligned} \pi(k_i + K_{-i}) &= c(k_i + K_{-i})^{-1}\left(\frac{p(k_i + K_{-i})}{2}\right)^2, \\ M(k_i + K_{-i}) &= p(k_i + K_{-i})(e - \bar{e}). \end{aligned}$$

Then, (6) becomes

$$\tilde{V}^i = \frac{\theta^{-1}\pi(k_i + K_{-i})}{r - \alpha - \sigma^2} - \frac{M(k_i + K_{-i})}{r} + A(k_i + K_{-i})\theta^{\beta^v}.$$

By focusing on a symmetric Nash equilibrium, dimensionality can be further reduced. Recall that in a symmetric equilibrium, $k_i = K/n$ for all i. Hence, by change of variables $c(k_i + K_{-i}) = \tilde{c}(K)$ and $\partial c(k_i + K_{-i})/\partial k_i = (\partial\tilde{c}(K)/\partial K)(\partial K/\partial k_i) = n\partial\tilde{c}(K)/\partial K$. Similarly, $p(k_i + K_{-i}) = \widetilde{p}(K)$, $\partial p(k_i + K_{-i})/\partial k_i = n\partial\widetilde{p}(K)/\partial K$, $A(k_i + K_{-i}) = \tilde{A}(K)$ and $\partial A(k_i + K_{-i})/\partial k_i = n\partial\tilde{A}(K)/\partial K$. Using this property, the value-matching condition (13) can be solved as

$$n\tilde{A}'\tilde{\theta}^{*\beta^v} = \frac{-\tilde{\theta}^{*-1}\widetilde{\pi}'(K)n}{r - \alpha - \sigma^2} + \frac{\widetilde{M}'(K)n}{r} + w \tag{28}$$

where $\widetilde{\pi}'(K) = \tilde{c}^{-1}\left(\frac{\widetilde{p}}{2}\right)^2\left[2\frac{\widetilde{p}'}{\widetilde{P}} - \frac{\tilde{c}'}{\tilde{c}}\right]$ and $\widetilde{M}'(K) = \frac{\tilde{P}'(e-\bar{e})}{r}$. For notational brevity, argument K is eliminated in function $\tilde{A}, \tilde{c}$ and $\tilde{P}$, and superscript prime denotes first derivative with respect to K. From the smooth-pasting condition,

$$\beta^v n\tilde{A}'\tilde{\theta}^{*\beta^v} = \frac{\tilde{\theta}^{*-1}\widetilde{\pi}'(K)n}{r - \alpha - \sigma^2}. \tag{29}$$

With $\varsigma_c = -\tilde{c}' K/\tilde{c}$ and $\varsigma_p = -\widetilde{p}' K/\widetilde{p}$, I solve (28) and (29) to obtain:

$$\tilde{\theta}^* = \frac{\frac{\widetilde{p}(K)^2}{2\tilde{c}(K)} \left(\frac{1}{2}\varsigma_c - \varsigma_p\right) n}{\left(n\widetilde{M}'(k) + w\right) H^v (r - \alpha - \sigma^2) K} \tag{30a}$$

where $H^v = \beta^v / (\beta^v + 1) > 0$ and $\widetilde{M}'(k) = \widetilde{p}'(e - \bar{e})/r$. It is easily verified that if $n = 1$ and $\tilde{P}' = 0$, $\tilde{\theta}^*$ reduces to θ^*:

$$\tilde{\theta}^* = \theta^* = \left(\frac{\widetilde{p}}{2\tilde{c}}\right)^2 \frac{-\tilde{c}'}{wH^v(r - \alpha - \sigma^2)}.$$

Q.E.D. ■

It can be shown that $n\widetilde{M}'(k) + w$ is the net investment cost defined by the capital adjustment cost (w) less the marginal change of discounted permit purchase cost ($n\widetilde{p}'(e - \bar{e})/r$). Note that $n\widetilde{M}'(k) + w < w$ since $\widetilde{p}' < 0$. In the analysis, I disregard a case when $n\widetilde{M}'(k) + w < 0$ because it is highly unlikely that investment cost becomes negative through incremental adjustment of abatement capital. Hence, Proposition 6 shows that $\tilde{\theta}^*$ remains positive if and only if the elasticity of permit price is substantially less than the elasticity of abatement cost adjusted by 1/2.[1] Intuitively the proposition implies that when costly investment is made, the decrease in abatement cost must be larger than the decrease in permit price. By contrary, suppose $\frac{1}{2}\varsigma_c < \varsigma_p$. Then, firms anticipate significant decrease in permit price accomplished by other firms' investments and, as a result, they may never invest while just waiting to buy permits.

A limiting property associated with (27) provides

$$\lim_{n \to 1} \lim_{p'=0} \tilde{\theta}^* = \theta^*$$

proving that, in the absence of price effect, the investment threshold $\tilde{\theta}^*$ converges to θ^*, the optimal investment threshold in the single firm case that is determined by monopolistic investment opportunity. On the other hand, the optimal investment threshold in perfect competitive equilibrium is given by

$$\lim_{n \to \infty} \lim_{p'=0} \tilde{\theta}^* = \infty$$

[1] Recall from Chapter 3 that the variable abatement cost function is given by $\theta c(k) a^\gamma$ and I set $\gamma = 2$. Actually, the denominator 2 indeed represents γ.

indicating that immediate investment is always better than waiting. This is because, in the absence of price effect, increasing number of firms lowers abatement cost through aggregate stock of abatement capital and hence facilitates investments among firms.

Lastly, the effect of competitiveness is characterized as follows:

Corollary *If the investment threshold exists under a TPP, its competitiveness effect is characterized as* $\frac{\partial \tilde{\theta}^*}{\partial n} > 0$.

The proof follows immediately from the derivative of (27) with respect to n. The corollary states that when the number of competiting firms grows, the level of optimal investment threshold for oligopolistic firms increases. This is intuitively explained by preemptive incentive for firms to take better position to sell their residual permits prior to other firms' sales.

3 Conclusions

We extended a single firm's investment option model to multifirms model to incorporate strategic option valuation. Firms interact each by participating emission permit market and contributing to aggregate abatement capital. Permit price and abatement cost are assumed to be influenced by aggregate capital. The result shows that, after modifying competitive equilibrium model to myopic setting, investment threshold in oligopolistic permit market can be derived for monopolistic investment opportunity. The derived investment thresholds are shown to converge to single firm's threshold as competition and market interaction vanish.

The model result indicates that when the investment is costly, the condition for existence of threshold requires sufficient cost reduction through the investment. If cost reduction effect is less than permit price reduction by investment, firms will never undertake investment because other firms' investment will eventually lower permit price more than abatement cost. Hence, firms can comply with the emission cap by purchasing permit price at lowered permit price but do not have incentives to increase abatement capital. The effect of advance allocation on investment was briefly discussed. The result shows that banking encourages more investment than non-banking system when there is no advance allocation, because through investment firms can retain more banked permits that can be used for future compliance or selling.

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