STRATEGIC BEHAVIOR AND MARK-UPS IN AN ELECTRICITY MARKET

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1. Introduction

Our main purpose of this study is to observe "mark-ups" in electricity markets such as PJM or Nord Pool, and evaluate their market performances. To address this issue, an oligopoly model is developed to explore the relation between strategic behavior of power generators and spot prices in an electricity market. It is an extent ion of the model presented in Tezuka and Ishii (2011). Other extensions are introduced by Ishii and Tezuka (2008a, b).

The outline of our model is as following. It is assumed that n homogeneous power generating firms are on the selling side of the transaction and the perfectly competitive many retailers are on the buying side. The retailers supply electricity to their customers. All power generating firms are supposed to select and offer exponential supply functions strategically under uncertain demand. Any coalitions among the firms are not assumed in this model. Then, the model is a non-cooperative game. The unique Nash equilibrium can be derived explicitly.

The paper is organized as follows: We briefly introduce our model in section 2. A non-cooperative game is explained and a unique Nash equilibrium is provided in section 3. Lastly, we interpret the properties of the equilibrium and conclude our research in section 4.

2. The Model

We adopt a non-cooperative game to describe an oligopolistic electricity market. For simplicity of the model, we use a one-period model. However, it is easy to extend our idea to a multi-period model. The method is described in Ishii (2007).In this section, we briefly describe our model.

We assume an oligopolistic spot electricity market, so there are a finite number of power generating firms and many perfectly competitive retailers. In the market, the power generating firms are suppliers, and face spot electricity demand by all retailers, who distribute power to their customers.

For simplicity, our model is one-period. We call the beginning of the period *time zero* and the end of the period *time one*. At *time zero*, quantity of spot electricity demand is uncertain, but each power producer has information on the distribution (see Fig.1.). Therefore each supplier strategically selects and offers a supply function.

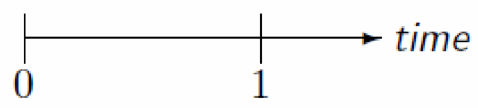

Fig.1. the timeline

After that, the market maker adds individual supply functions to construct a market supply function. At *time one*, the demand will be realized, and the market maker will decide on a spot price with the market supply function and the realized demand. Here, it is worth noting that we can easily extend our approach to a multi-period model by the method described in Ishii(2007).

Let $(\Omega, \mathscr{F}, P)$ be a probability space. A non-negative continuous random variable X is called the total electricity demand at time one. At *time zero*, they know only the probability distribution of X, but the value is unforeseeable for all market participants.

$n \geq 2$ denotes the number of power generating firms. Suppose a and b are positive constraints. We set:

$$f_j = e^{ax+b} \qquad for \quad x \in [0, \infty)$$

where f is the marginal cost function of each power generating firm j (j=1,2,…..,n). We use an exponential function as the marginal cost function.

We also define $g_j: [0,\infty)^2 \to [0,\infty)$ by

$$g_j(x, s_j) = e^{ax+b+s_j} \qquad for \quad x \in [0,\infty)^2$$

where $s_j \in A_j$. $A_j = [0,\infty)$ is strategy set for power generator *j*. We call g_j a supply function that a power producer *j* offers to the market. Note that g_j is not same as the marginal cost function f_j.if strategies ("mark-ups") exist, or are positive value.

For each *j=1,2,…,n*, $s_j \in A_j$ is a strategy of power generator *j*. With the distribution of *X* and information about the other All strategies are shifts of the marginal cost function. Each generating firm decides its strategy at the beginning of the period. Ishii and Tezuka(2011) assumed the liner marginal cost function. In this model, we assume an exponential marginal cost function.

All consumers' demands are aggregated through the retailers to the market electricity demand. Therefore, the total demand is assumed to realize at the end of the period. On the other hand, all power generating firms can not exactly foresee the total demand, which is a random variable, but know the distribution function at the beginning of the period.

Similar to Ishii and Tezuka(2011) in the setting of liner marginal/supply cost functions, we derive the spot price formulae under the setting.

$$\varphi(x,s) = \begin{cases} e^{ax+b+s_{(1)}} & \text{for } x \in I_1(s) \\ e^{\frac{1}{k}\left(ax+bk+\sum_{i=1}^{k} s_{(i)}\right)} & \text{for } k = 2,3,\ldots,n-1, \text{and} \\ & x \in I_k(s) \\ e^{\frac{1}{n}\left(ax+bn+\sum_{i=1}^{n} s_i\right)} & \text{for } x \in I_n(s) \end{cases}$$

Then, we also derive the profit function of firm1:$F_1(x,s)$ by using the spot price formula.

1)For the case of $s_1 = s_{(1)}$ and $x \in I_1(s)$,

$$F_1(x,s) = \frac{e^{ax+b+s_1}}{a}(ax - e^{-s_1}) + \frac{e^b}{a}$$

2)For the case of l=1,2,….,n, $s_1 = s_{(1)}$ and $x \in \bigcup_{i=1}^{l} I_i\,(s)$,

$$F_1(x,s) = 0$$

3)For the case of k=l+1, l+2,……,n-1, $s_1 = s_{(1)}$ and $x \in I_k(s)$,

$$F_1(x,s) = \frac{e^{\frac{ax+bk+\sum_{i=1}^{k} s_{(i)}}{k}}}{a} \times \left(\frac{ax-(k-1)s_1+\sum_{1\le i\le k, i\ne l} s_{(i)}}{k} - e^{-s_1} \right) + \frac{e^b}{a}$$

4)For the case of $x \in I_k(s)$,

$$F_1(x,s) = \frac{e^{\frac{ax+bk+\sum_{i=1}^{n} s_i}{n}}}{a} \times \left(\frac{ax-(n-1)s_1+\sum_{i=2}^{n} s_i}{n} - e^{-s_1} \right) + \frac{e^b}{a}$$

We can easily derive $F_2(x,s)$, $F_3(x,s)$,…………., $F_n(x,s)$ in similar way.

3. Nash Equilibrium

We derived the unique Nash equilibrium in an explicit form where n power generating firms have homogeneous exponential supply functions. To address this, we set following non-cooperative game: For $\alpha\epsilon(0,1)$,

$$\begin{cases} \text{the strategy set for player } j \text{ is } [0,\infty) \\ \text{the payoff to player } j \text{ is } F_j(x_\alpha) \end{cases} \qquad (*)$$

Note that x_α denotes the α-quantile of demand distribution of X. The payoff function has some superior properties[1]. The higher α means that each power producer is risk lover and vice versa.

Lemma

In the non-cooperative game(*), if there exists m=1,2,…,*n* such that $F_m(x_\alpha, s) = 0$, then $s = (s_1, s_2, \ldots\ldots, s_n) \in [0, \infty)^n$ is not Nash equilibrium.

By using this Lemma, if there exist Nash equilibriums in above game, they are included

$$\left\{ s \in [0,\infty)^n \;\middle|\; F_j(x_\alpha, s) > 0 \text{ for } \forall j = 1,2,\ldots,n \right\}$$
$$= \left\{ s \in [0,\infty)^n \;\middle|\; \sum_{i=1}^{n-1} (s_{(n)} - s_{(i)}) < a x_\alpha \right\}$$

Then we can obtain the following Theorem:

Theorem

In the non-cooperative game (*), there exist Nash equilibriums if and only if

$$1 > \frac{a x_\alpha}{n(n-1)}$$

Under the conditions, there exists the following unique Nash equilibrium.

$$s_j{}^* = -\log\left(1 - \frac{a x_\alpha}{n(n-1)}\right) \tag{1}$$

The equilibrium strategy shows the degrees of "mark-ups" that the power producer *j* offers to the market.

Then, we obtain the equilibrium spot price:

$$\varphi(X, s^*) = \frac{e^{\frac{aX}{n}+b}}{1 - \frac{a x_\alpha}{n(n-1)}}. \tag{2}$$

Next, we interpreted the properties of the equilibrium.

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1 See Ishii(2007) or Ishii and Tezuka(2011) in detail.

4. Interpretations from the equilibrium and Conclusion

We derived the unique Nash equilibrium in an explicit form. Regarding mark-ups, we find some interpretations by using these results.

First, by the equations (1) and (2), if the number of generating firm *n* is large enough, the mark-up approaches to zero. In the case, each power producer offers the supply curve close to marginal cost. Conversely, if *n* is small, the mark-up becomes low. The result is consistent with the SCP paradigm. Second, the equilibrium spot price is an increasing function of power generators' risk attitude. The lower α, which means every power generating firm is risk averse, results in lower mark-ups. Third, other things being equal, the higher elasticity of supply, which denotes lower *a* in (1) and (2), is higher, the mark up become lower and vice versa.

In this paper, the relation between the supply function strategies and the spot price process is revealed under some conditions. However, to confirm applicability of our model, empirical analysis is necessary for our future research.

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REFERENCES

Ishii, M. (2007), ``The Basis for a Game-theoretic Analysis of Wholesale Electricity Markets", *Management Journal*(Daito Bunka University), Vol.14, pp.31-42.

Ishii, M. and Tezuka, K.(2011), ``Assessing the Impact of Strategic Behavior on Spot Prices in an Electricity Market ", *Proceedings of 30th USAEE/IAEE North American Conference*, Washington, DC, United States.

Ishii, M. and Tezuka, K.(2008a), ``A Study on a Non-cooperative Game-theoretic Pricing Framework in a Oligopolistic Electricity Market", *Proceedings of 31st Annual IAEE International Conference*, Istanbul, Turkey.

Ishii, M. and Tezuka, K.(2008b), ``A Study on an Asset Valuation Framework for Electricity Markets", *Proceedings of 2nd IAEE Asian Conference*, Taipei, Taiwan.

Klemperer, P., D., and Meyer, M. A.(1989), ``Supply Function Equilibria in Oligopoly under Uncertainty," *Econometrica*, Vol. 57, pp. 1243-1277.

Tezuka, K., and Ishii, M.(2007), ``A Game Theoretical Analysis of the Spot Prices in Wholesale Electricity Markets," *Proceedings of 30th Annual IAEE International Conference,* Wellington, New Zealand.