

*On Random Walk hypothesis for the Crude Oil, Bituminous Coal  
and Natural Gas Markets: Evidence from Regime Switching  
Approach*

Shyh-Wei Chen<sup>1</sup>

Department of International Business  
Center for Applied Economic Modeling  
Chung Yuan Christian University  
E-mail: shyhwei.chen@gmail.com or  
shyhwei.chen@cycu.edu.tw

Shih-Mo Lin

Department of International Business  
Center for Applied Economic Modeling  
Chung Yuan Christian University  
E-mail: shihmolin@gmail.com

Paper prepared for the 3rd IAEE Conference

<sup>1</sup>Corresponding author. Department of International Business, Center for Applied Economic Modeling, Chung Yuan Christian University. No. 200, Chung Pei Rd, Chung Li, Taiwan 32023, ROC. Tel: 886-3-2655213, Fax: 886-3-2655299. E-mail: shyhwei.chen@gmail.com or shyhwei.chen@cycu.edu.tw. This is a short version of our paper entitled *On Non-stationarity and Non-linearity of the Non-renewable Resource Prices: Evidence from the Regime Switching Approach*.

## **Abstract**

This paper examines the non-stationary and non-linear features of the non-renewable resource markets: the crude oil (the US West Texas Intermediate and the UK Brent), bituminous coal and natural gas. In particular, we achieve this goal by using the Markov switching unit root regression. This approach is attractive because it allows price to switch between stationary and non-stationary regimes (partial non-stationarity). It also allows price to switch between two stationary regimes (varied stationarity) or to switch between two non-stationary regimes. The results of a range of non-linear tests show that the independently and identically distributed (i.i.d.) hypothesis or the random walk hypothesis is untenable for the non-renewable resource prices. The results from Markov regression indicate that, in the cases of the US West Texas Intermediate, the UK Brent as well as bituminous coal, prices are characterized by the local non-stationarity in both regimes, and therefore non-stationarity sustains. For price of natural gas, it is characterized by partial non-stationarity. The posterior probabilities stem from the Markov switching unit root regression are discussed in the text.

**Keywords:** Random walk, crude oil, bituminous coal, natural gas, Markov switching

**JEL classification:** C22, Q400

# 1 Introduction

Testing for a unit root or the non-stationarity hypothesis in asset prices has attracted substantial interest in the empirical finance literature ever since the studies of Fama and French (1988a, 1988b), Lo and MacKinlay (1988) and Poterba and Summers (1988) were published. The reason for this is that if there is a unit root in the asset price and error term is assumed to be an independent and identical distribution (hereafter i.i.d.), then it implies that asset returns cannot be predicted by previous price changes.<sup>1</sup> Therefore, given only past price and return data, the current price is the best predictor of the future price, and the price change or return is expected to be zero. This is the essence of the weak-form efficient markets hypothesis (hereafter WEMH). However, if asset prices follow a mean reverting or stationary process, then there exists a tendency for the price level to return to its trend path over time and investors may be able to forecast future returns by using information on past returns.

In natural resource markets, examining whether price movements exhibit a unit root and forecasting the future level of prices and their fluctuations have also attracted substantial research over the past two decades. Natural gas, historically a substitute for refined petroleum products, is considered to have a price movement closely aligned to that of crude oil. Although oil and natural gas prices might exhibit seemingly independent movements, a long-run linear relationship still exists between them. For example, Villar and Joutz (2006), Asche et al. (2006), and Brown and Yücel (2008) all found that the oil price and natural gas price are cointegrated. Brown and Yücel (2008), however, indicate that the cointegrated relationship between oil and natural gas prices is conditioned by weather, seasonality, etc. Coal prices, by contrast, are less volatile as compared to those of crude oil and natural gas. Nonetheless, the movement of coal prices is found to follow a stationary process (Lee et al., 2006).

Three important, albeit somewhat disappointing, features characterize previous studies on the unit root hypothesis. The findings are mixed depending on the different markets, frequency, time period and methodologies employed in previous studies, which means there is no corroborative conclusion vis-à-vis the stationarity property for natural resource prices. Second, the majority of these earlier studies apply the traditional method in testing for the null hypothesis of a unit root of the asset prices. It is well-known that the traditional unit root test is powerless if the true data generating process of a series exhibits structural breaks (Perron, 1989). Therefore, a few of the studies, e.g., Lee et al. (2006), Postali and Picchetti (2006), Maslyuk and Smyth (2008) and Narayan

---

<sup>1</sup>Rahman and Saadi (2008) emphasize the difference between unit root test and the random walk hypothesis. Tests for the random walk hypothesis are concerned with the predictability of future price changes, which explains the need for i.i.d. assumption. Unit root tests are designed to investigate whether a series is difference-stationary or trend-stationary. Although share non-stationarity is a necessary prerequisite for the random walk hypothesis, it is not a sufficient condition. That is to say, the random walk hypothesis is equivalent to the combination of the unit root and i.i.d. assumption.

et al. (2008) adopt newly-developed unit root tests with structural breaks (Lee and Strazicich, 2003a, 2003b) to investigate the stationarity property of natural resource prices. Third, Recent studies, e.g., Ewing et al. (2002), Kyrtsou et al. (2009) and Maslyuk and Smyth (2009), point out that non-renewable resource prices should be specified as non-linear data generating processes, implying that the conventional unit root test, which assume linear and systematic adjustment with respect to stationarity, are misleading and indicating that the reliability of the findings from existing studies is questionable.<sup>2</sup>

Testing for non-linearity or non-linear dependence for natural resource prices are of importance. If the non-linearity really exist in natural resource price, then this feature should be taken into account in model specification at least. If not, it is expected to cause serious bias in forecasting. In addition, the nonlinear feature of natural resource price has different impact on macroeconomic variables through transmission mechanism. However, testing for the null hypothesis of the non-stationarity or the non-linearity of an asset price goes astray in different avenue in the literature. When testing for the non-stationarity of a series, the conventional linear unit root statistic always ignores the property of the non-linearity and vice versa. This is because testing for the properties of the non-stationarity and non-linearity of a series simultaneously involves tedious non-standard distribution theory. This difficulty make analysts to avoid the problem by testing the non-stationarity and non-linearity one by one.

The central aim of this paper is to examine the non-stationarity and non-linearity of the non-renewable resource markets, i.e., the prices of the US West Texas Intermediate (WTI) and the UK Brent crude oils, bituminous coal and natural gas. A key contribution of this research is that we tackle the non-stationarity and non-linearity simultaneously based on the augmented Dickey-Fuller (ADF) unit root testing within a Markov regime-switching framework. The Markov switching approach is in sharp contrast to existing studies of testing non-linearity and offers valuable new insights into price behavior. These non-renewable resource prices are modeled as regime dependent where episodes of stationarity or non-stationarity can be identified and analyzed. In contrast, most existing studies compute a single test statistic for testing non-stationarity across the entire study period. This approach can lead to a bias towards accepting the non-stationary null, because there is no distinction between alternative regimes.

The remainder of this paper is organized as follows. Section 2 introduces the econometric methodology that we employ, and Section 3 describes the data and the empirical test results. Section 4 presents the conclusions that we draw from this research.

---

<sup>2</sup>For the benefit of readers, we summarize the recent contributions for testing non-stationarity and non-linearity of natural resource prices after 2000 in Table 1. Readers are referred to reference cited in Lee et al. (2006) and Maslyuk and Smyth (2009) for more studies published in pre-2000.

Table 1: Selective papers on non-stationarity and non-linearity for natural resources after 2000

Studies	Energy and Samples covered	Methodology	Results
Krichene (2002)	natural gas and crude oil 1918 to 1999	ADF unit root test	unit root is acceptable
Abosedra (2005)	crude oil spot and future prices January 1991 to December 2001	ADF unit root test and Phillips and Loretan (1991) non-linear cointegration estimation	spot and future prices have a unit root future price tends to be semi-strongly efficient
Lee et al. (2006)	11 natural resource annual data 1870–1990	Lee and Strazicih LM unit root test with breaks	coal and gas are stationary process with breaks
Postali and Picchetti (2006)	annual oil price 1861–1999	Lee and Strazicih LM unit root test with breaks	oil price is stationary process with breaks
Maslyuk and Smyth (2008)	WTI and Brent crude oil spot and future prices, 1991–2004	Lee and Strazicih LM unit root test with breaks	WTI and Brent oil prices are characterized by a unit root with breaks
Maslyuk and Smyth (2009)	crude oil for 17 OPEC and non-OPEC countries, 1973–2007	Caner and Hansen (2001) non-linear unit root test	crude oil is characterized threshold effect and 11 of 17 has unit root
Tabak and Cajueiro (2007)	WTI and Brent crude daily oil prices 1983–2004	time-varying long-range dependence	WTI and Brent oil markets are weak-form efficient market
Alvarez-Ramirez et al. (2008)	Brent, WTI and Dubai crude oil prices 1987–2007	Hurst exponent dynamics (detrended fluctuation analysis)	Brent, WTI and Dubai crude oil markets are weak-form efficient market
Charles and Darné (2009)	WTI and Brent crude oil daily price, 1982–2008	a range of variance ratio tests	Brent is WEMH WTI is not WEMH in 1994 to 2008
Lean et al. (2010)	the WTI crude oil daily January 1, 1989 to June 30, 2008	mean-variance (MV) and stochastic dominance (SD)	the spot and futures oil markets are efficient
Alvarez-Ramirez et al. (2010)	the WTI crude oil daily January 1, 19896 to December 31, 2009	lagged detrended fluctuation analysis (DFA)	the WTI oil price is not an efficient market

## 2 Methodology

Let  $q_t$  denote the logarithm of the, e.g. price of crude oil. The Markov Switching ADF (MS-ADF) regression is obtained by running the following regression:

$$\Delta q_t = \alpha_{S_t} + \phi_{S_t} q_{t-1} + \sum_{j=1}^k \gamma_{j,S_t} \Delta q_{t-j} + u_t, \quad (1)$$

$$u_t \sim NID(0, \sigma_{S_t}^2), \quad (2)$$

where  $\Delta q_t$  denotes the first difference of the crude oil price  $q_t$ .  $\mu_{S_t}$ ,  $\phi_{S_t}$  and  $\gamma_{1,S_t}, \dots, \gamma_{k,S_t}$  are regime-varying parameters, and  $u_t$  is the innovation process with a regime-dependent variance-covariance matrix  $\sigma_{S_t}^2$ . The unobservable state variable  $S_t$  follows a first-order, two-state Markov Chain with the transition probability as follows:

$$\text{pr}(S_t = j | S_{t-1} = i) = p_{ij}, \quad (3)$$

where  $i, j = 0$  or  $1$ . The unconditional probabilities for state 1 and state 2 are  $w_1 = \frac{1-p_{22}}{2-p_{11}-p_{22}}$  and  $w_2 = \frac{1-p_{11}}{2-p_{11}-p_{22}}$ , respectively.

An interesting feature of this model is that no assumption is needed to impose the stationarity of either regime. Local stationarity in both regimes is confirmed if the null  $\phi_i = 0$ ,  $i = 0, 1$  is rejected. Being conditional on this, if  $\phi_0 \neq \phi_1$ , we may define the concept of *varied stationarity* because stationarity is confirmed across the entire study period, but the autoregressive coefficients and speeds of adjustment towards long-run equilibrium are different.<sup>3</sup> If the hypothesis of  $\phi_i = 0$ ,  $i = 0, 1$ , cannot be rejected, then it is indicative of local non-stationarity in both regimes, and therefore non-stationarity sustains. However, we might only be able to confirm that either  $\phi_0$  or  $\phi_1$  is insignificantly different from zero. In this case, we may define the concept of *partial non-stationarity* or *partial unit root* because the price of crude oil is switching between regimes of stationarity and non-stationarity.

A useful information of model (1)–(2), in our case, is that it enables us to identify sub-periods during which the price of crude oil seems to be stationary and treat these periods as a signal: The longer the economy stays in these periods, the more likely it is that the non-stationarity will not hold. This econometric methodology allows us to distinguish periods that are associated with stationary outcomes from those in which the non-stationarity holds.

---

<sup>3</sup>If  $-1 < \phi_0, \phi_1 < 0$ , the half-life associated with a deviation from long-run equilibrium may be approximated as  $HL_0 = (\ln 0.5)/(1 + \phi_0)$  and  $HL_1 = (\ln 0.5)/(1 + \phi_1)$  for Regimes 0 and 1 respectively.

### 3 Data and Results

We use the prices of two crude oil markets: the US West Texas Intermediate and the UK Brent. The data comes from Thomson Financial Datastream and is given in US dollar per barrel. The data spans from January, 1982 to July, 2011, namely 355 observations. We also consider prices of bituminous coal and natural gas in our empirical study. Monthly price series ranging from January 1997 to September 2010 for bituminous coal, and from January 1976 to June 2011 for natural gas are used in the current paper. Log transformations for these non-renewable resource prices are used throughout the study.

The first stage of the empirical investigation is to test for the non-stationarity of these non-renewable resource prices. Basically, we find no additional evidence against the unit root hypothesis at the 5% significance level based on the ADF, PP, DF-GLS and NP tests in their level data. When we apply the ADF, DF-GLS and NP tests to the first difference of these series (available from the author upon request), we must reject the null hypothesis of a unit root at the 5% level or better. This implies that prices of two crude oil markets: the US West Texas Intermediate and the UK Brent, as well as prices of bituminous coal and natural gas are difference-stationary processes.

We now consider the possibility that the presence of a unit root, or the inability to reject the null of a non-stationary prices, may be attributable to hitherto unacknowledged regime switches with respect to the time-series properties of the data. In order to validate the Markov switching model used in this paper, we first conduct several nonlinearity tests for these non-renewable resource prices. Psaradakis and Spagnolo (2002) examines the relative performance of some popular nonlinearity tests when applied to time series generated by the Markov switching autoregressive models. The nonlinearity tests considered include RESET-type tests, the Keenan test, the Tsay test, the McLeod-Li test, the BDS test, the White dynamic information matrix test, and the neural network test.<sup>4</sup> We adopt these statistics to examine whether there any nonlinearity exists in these non-renewable resource prices. The results are reported in Table 2. Table 2 shows that most of the  $p$ -values of these nonlinear tests are smaller than the 5% significance level. For example, the simulation results of Psaradakis and Spagnolo (2002) indicate that the BDS test has the highest power performance relative to other nonlinear statistics. We observe that the  $p$ -values of the BDS statistics are smaller than the 5% significance level or better, indicating that the prices of the Brent oil, WTI oil, bituminous coal and natural gas are not i.i.d., indicating that these non-renewable resource prices do not follow the random walk process. They are better characterized by alternative nonlinear model such as the Markov switching model.

Having established the ‘global’ and ‘nonlinear’ characteristics of the series, we now focus on their ‘local’ behavior by estimating the Markov switching models discussed in Section 2. This approach has the advantage of neither splitting the sample period into different sub-periods nor

---

<sup>4</sup>Readers are referred to Psaradakis and Spagnolo (2002) for detailed descriptions of these tests.

Table 2:  $p$ -values for a battery of non-linear tests

	Brent	WTI	Coal	Gas
RESET1	0.024	0.031	0.020	0.000
RESET2	0.024	0.031	0.020	0.003
KEENAN	0.638	0.121	0.052	0.814
TSAY	0.637	0.121	0.052	0.035
MCLEOD	0.011	0.000	0.000	0.000
BDS	0.000	0.000	0.002	0.000
WHITE1	0.341	0.333	0.215	0.752
WHITE2	0.000	0.003	0.157	0.000
NEURAL1	0.012	0.030	0.041	0.000
NEURAL2	0.049	0.046	0.040	0.089

(1) RESET1: Ramsey and Schmidt (1976). (2) RESET 2: Thursby and Schmidt (1977). (3) KEENAN: Keenan (1985). TSAY: Tsay (1986). (4) The Ramsey-Schmidt test is referred to as RESET1. (5) The Thursby-Schmidt test is referred to as RESET2. (6) MCLEOD: McLeod and Li (1983). (7) BDS: Brock et al. (1996). (8) WHITE1 and WHITE2 are White's (1987) information matrix tests. (9) NEURAL1 is the neural network test proposed by White (1989a). (10) NEURAL2 is the neural network test proposed by White (1989b).

pre-imposing regime dates. In Table 3 we report maximum likelihood (ML) estimates (based on the Gaussian likelihood) and associated asymptotic standard errors of the parameters of the Markov switching ADF Eqs. (1)–(2) for these non-renewable resource prices. The estimated value of  $\sigma_1$  is substantially larger than that of  $\sigma_0$ , and thus regime 1 corresponds to the high-volatility regime while regime 0 corresponds to the low-volatility regime. The rejection of the null hypothesis of  $H_0^\sigma : \sigma_0 = \sigma_1$  throughout is consistent with the two regimes being characterised by different volatilities.

Before reporting the estimated results, it is worthy to mention the simulation results reported in Kanas and Genius (2005). They assess the power of the standard ADF test when the data generation process is characterized by regime switching in volatility. They provide simulation evidence that the when the autoregressive coefficient in the stationary regime is close to zero, the MS-ADF test always rejects the zero non-stationary null, whereas the standard ADF unit root test fails to do so in two-fifths of the cases. Although the power of the ADF test increases as the autoregressive coefficient in the stationary regime moves further away from zero, the MS-ADF still maintains an advantage in terms of test power.

In the cases of prices of the US West Texas Intermediate and the UK Brent, as well as bituminous coal, in regime 0 (the low-volatility regime), the ADF statistics fail to reject the null hypothesis of the non-stationarity ( $\phi_0 = 0$ ) at the 5% significant level.<sup>5</sup> In regime 1 (the high-volatility regime), the ADF statistics also fail to reject the null hypothesis of a unit root ( $\phi_1 = 0$ ) at the 5%

<sup>5</sup>Gabriela et al. (2002) show that using standard critical values for unit root testing is acceptable when testing for cointegration in two steps when using data generated from a two regimes model.



Table 3: Estimation Results of the Markov Switching Unit Root Regression

	Brent	WTI	Coal	Gas
$\alpha_0$	-0.045 (0.027)	-0.022 (0.025)	0.059 (0.025)	0.011 (0.002)
$\alpha_1$	0.289 (0.106)	0.248 (0.116)	0.102 (0.093)	0.029 (0.014)
$\sigma_0$	0.062 (0.004)	0.062 (0.003)	0.025 (0.003)	0.021 (0.001)
$\sigma_1$	0.124 (0.013)	0.135 (0.018)	0.093 (0.009)	0.107 (0.004)
$p_{00}$	0.863 (0.096)	0.850 (0.072)	0.830 (0.096)	0.997 (0.003)
$p_{11}$	0.960 (0.026)	0.976 (0.015)	0.881 (0.057)	0.996 (0.006)
$\phi_0$	0.016 (0.008)	0.008 (0.007)	-0.017 (0.007)	-0.013 (0.003)
$\phi_1$	-0.096 (0.034)	-0.077 (0.035)	-0.021 (0.022)	-0.024 (0.011)
$\gamma_0$	0.044 (0.069)	0.099 (0.061)	0.580 (0.064)	0.284 (0.077)
$\gamma_1$	0.360 (0.110)	0.470 (0.136)	0.400 (0.134)	0.188 (0.058)
$t_{\phi_0}$	2.000	1.142	-2.428	-4.333
$t_{\phi_1}$	-2.820	-2.200	-0.954	-2.181
$H_0^c$	0.000	0.000	0.000	0.000
RCM	38.997	26.579	41.673	5.684
LL	-395.678	-420.293	-252.258	-565.604

$S_t = 0$  is the local stationary (or non-stationary) with low-volatility regime.  $S_t = 1$  is the local non-stationary with high-volatility regime.  $H_0^c$  refers to the null hypothesis  $\sigma_0 = \sigma_1$ .  $H_0^B$  refers to the null hypothesis  $\phi_0 = \phi_1$ . Figure for  $t_{\phi_0}$  is  $t$ -statistic for the null hypothesis  $\phi_0 = 0$ . Figure for  $t_{\phi_1}$  is  $t$ -statistic for the null hypothesis  $\phi_1 = 0$ . Figures for  $H_0^c$  are  $p$ -values. Figures in parentheses are standard errors. RCM is the abbreviation of regime classification measure.

significant level. These findings are indicative of the local non-stationarity in both regimes, and therefore non-stationarity sustains.

The case where stationarity is present in one regime only includes the price of natural gas. The simple unit root test shows that we reject the null hypothesis of  $\phi_0 = 0$  at the 5% significant level ( $t_{\phi_0} = -4.333$ ), while we cannot reject the null hypothesis of  $\phi_1 = 0$  at the 5% significant level ( $t_{\phi_1} = -2.181$ ), which is indicative of the fact that state 0 is the locally stationary with low-volatility regime and, of course, state 1 is the locally non-stationary with high-volatility regime for the price of natural gas. In other words, in case of natural gas which is characterized by partial non-stationarity.

The partial non-stationarity indicates that if the price of natural gas is staying in the stationary with low-volatility regime, then the price is predictable and investors or speculators will race to take advantage of it. That is, they will seize opportunity to arbitrage in order to make extra profit from the market. Those investors who spot the opportunity first and who trade quickly will have the ability to exploit it. However, prices will respond quickly to those buying and selling processes, causing the arbitrage opportunity to evaporate and finally switch into the non-stationary regime, and therefore partial non-stationarity sustains.

A merit of the Markov switching model is that for each time period, it allows the researcher to

Table 4: Duration periods for staying in stationary and non-stationary states

	Brent	WTI	Coal	Gas
$p_{00}$	0.863	0.850	0.830	0.997
$p_{11}$	0.960	0.976	0.881	0.996
$(1 - p_{00})^{-1}$	7.3	6.7	5.9	333
$(1 - p_{11})^{-1}$	25	41.7	8.4	250
Condition (A)	✓	✓	✓	
Non-stationarity hold	high	high	high	medium

$S_t = 0$  is the local stationary (or non-stationary) regime.  $S_t = 1$  is the local non-stationary regime. ✓ denotes the condition is satisfied. Condition (A): the expected periods of remaining in local non-stationary regime are longer than those of remaining in the local stationary regime. Non-stationarity hold: The likelihood of the non-stationarity holding.

estimate the posterior probability of the non-renewable resource price being subject to a particular (stationary or non-stationary) regime. The inferred probabilities that the Eqs. (1)–(2) are in the local non-stationary with high-volatility regime at each date for the prices of the Brent, WTI oil, bituminous coal and natural gas are shown in Figures 1, together with time series plots of the logarithm of these non-renewable resource prices under analysis. We use the 0.5 rule (Hamilton, 1989) to assign the observation at time  $t$  to state  $i$  if the filtered probability is larger than 0.5.

This paper aims to determine the likelihood of the non-stationarity to hold. If this is not true, then it signifies a signal that the price observed during the period are probably not an efficient market. We employ the following criterion (A): the expected periods of remaining in the local stationary regime are shorter than those of remaining in the local non-stationary regime. We can easily determine this condition by calculating the duration period using the formula  $(1 - p_{ii})^{-1}$ ,  $i = 0, 1$ , and observe the graph of the inferred probabilities. If this condition is accepted, then we expect that there will be a strong likelihood that the non-stationarity will hold. The results of this condition for these non-renewable resource prices are summarized in Table 4.

Starting with the Brent oil price (Figure 1), the dates of 1986, 1990–1991, 1998, 1999, 2001, 2002, 2003, 2008 are identified as the local non-stationary with high-volatility regime. For the case of WTI oil price (Figure 1), the dates of 1986, 1990–1991, 1999 and 2008 are also identified as the local non-stationary with high-volatility regime. William (2005) and Maslyuk and Smyth (2008) summarize some critical crude oil market chronologies, highlighting financial and political events that have impacted on oil prices. For instance, the excess supply of crude oil led to a crash of oil price to 10 US dollar per barrel by mid-1986. The Gulf War occurred in August 1990 caused a sudden rush of the oil price. The Asian financial crisis and Russian default occurred in 1997, which combined with increased OPEC production in 1998 ‘sent prices into a downward spiral’. Terrorist attack in 11 September 2001 was another critical event, which caused a plummet in both spot and futures oil prices. Political unrest in the first 6 months of 2002 associated with a series

of events in Venezuela caused oil price to rise and kept oil markets in a perpetual state of unrest. The second Gulf War occurred in March 2003 also caused a sudden sharp decline in oil prices. Recently, the so-called third crude oil crisis occurred when price reached its highest level of USD 145 per barrel at the middle of 2008. Afterward the price plunged to USD 30 per barrel at the end of 2008 due to speculations.

Visual inspection based on the posterior probabilities allows us to select periods during which price of non-renewable resource enter into stationary path and therefore non-stationarity is denied. The expected period for remaining in the regime 1 is 25 (41.7) months while the estimated expected period for remaining in regime 0 is around 7.3 (6.7) months for the Brent oil price (WTI oil price). Therefore, we expect the likelihood that the non-stationarity will hold to be high.

In the case of the price of bituminous coal, the MS-ADF test shows that we cannot reject the null hypothesis of a unit root either in regime 0 or regime 1. The corresponding filter probabilities (Figure 1) show that the high-volatility regime occurs more frequently than the low-volatility regime. The expected time for remaining in regime 0 is 5.9 months, while the estimated expected time for remaining in regime 1 is only around 8.4 months. We therefore expect that the likelihood that the non-stationarity will hold is high.

Moving to the natural gas market, the filtered probabilities (Figure 1) display a clear dichotomy in 1997. The graph of the filtered probabilities shows that the local stationary regime is associated with the period 1976–1997. After 1997, the regime switches to the local non-stationary regime and remains in this situation up to 2011. The expected times for remaining in the local stationary and non-stationary regimes are  $(1 - 0.997)^{-1} = 333$  and  $(1 - 0.996)^{-1} = 205$  quarters, respectively. Given that the last part of the sample is identified with a non-stationary period and condition (A) is violated, we conclude that the likelihood that the non-stationarity will hold is medium.

To assess the quality of regime switching in model (1)–(2), we calculate the regime classification measure (RCM) proposed by Ang and Bekaert (2002). It is defined as

$$RCM = 400 \times \frac{1}{T} \times \sum_t p_t(1 - p_t)$$

where  $p_t$  is the filtered probability of being in a certain regime at time  $t$ . Basically, this is a sample estimate of the variance of the probability series. It is based on the idea that perfect classification of regime would infer a value of 0 or 1 for the probability series and be a Bernoulli random variable. Good regime classification is associated with low RCM statistic values. A value of 0 means perfect regime classification and a value of 100 indicates that the two-regime model simply assigns each regime a 50% chance of occurrence throughout the sample. Weak regime inference implies that the model cannot successfully distinguish between regimes from the behavior of the data and may indicate misspecification. Consequently, a value of 50 is often used as a benchmark (Chan et al., 2011). The RCM measures are reported in the bottom of Table 3. It is not surprise to see that the RCM values for all the series are reasonably low, especially for the case of natural gas. This shows

that the model is able to confidently distinguish which regimes are occurring at each point in time.

## **4 Concluding Remarks**

This is the first study that tests for the properties of non-stationarity and non-linearity simultaneously for the non-renewable resource markets, i.e., the crude oil (the US West Texas Intermediate and the UK Brent), bituminous coal and natural gas, within a Markov regime-switching framework. Whereas for a battery of standard unit root tests involving for these non-renewable resource prices, non-stationarity is accepted in every case. Results from a range of non-linear test show that the non-renewable resource prices exhibit non-linear feature and therefore the i.i.d. or the random walk hypothesis does not sustain. The application of a Markov switching unit root test indicates that non-stationarity of natural gas price, and therefore non-stationarity, is present at least one regime. Rather than non-stationarity on the basis of a single test statistic across the entire study period, this result suggests that the natural gas price is characterized by partial non-stationarity, where the price shifts between local stationary and local non-stationary regimes. In the cases of the US West Texas Intermediate, the UK Brent and bituminous coal, price is characterized by the local non-stationarity in both regimes, and therefore non-stationarity sustains.

## **References**

The detail list of reference are available from the author upon request.

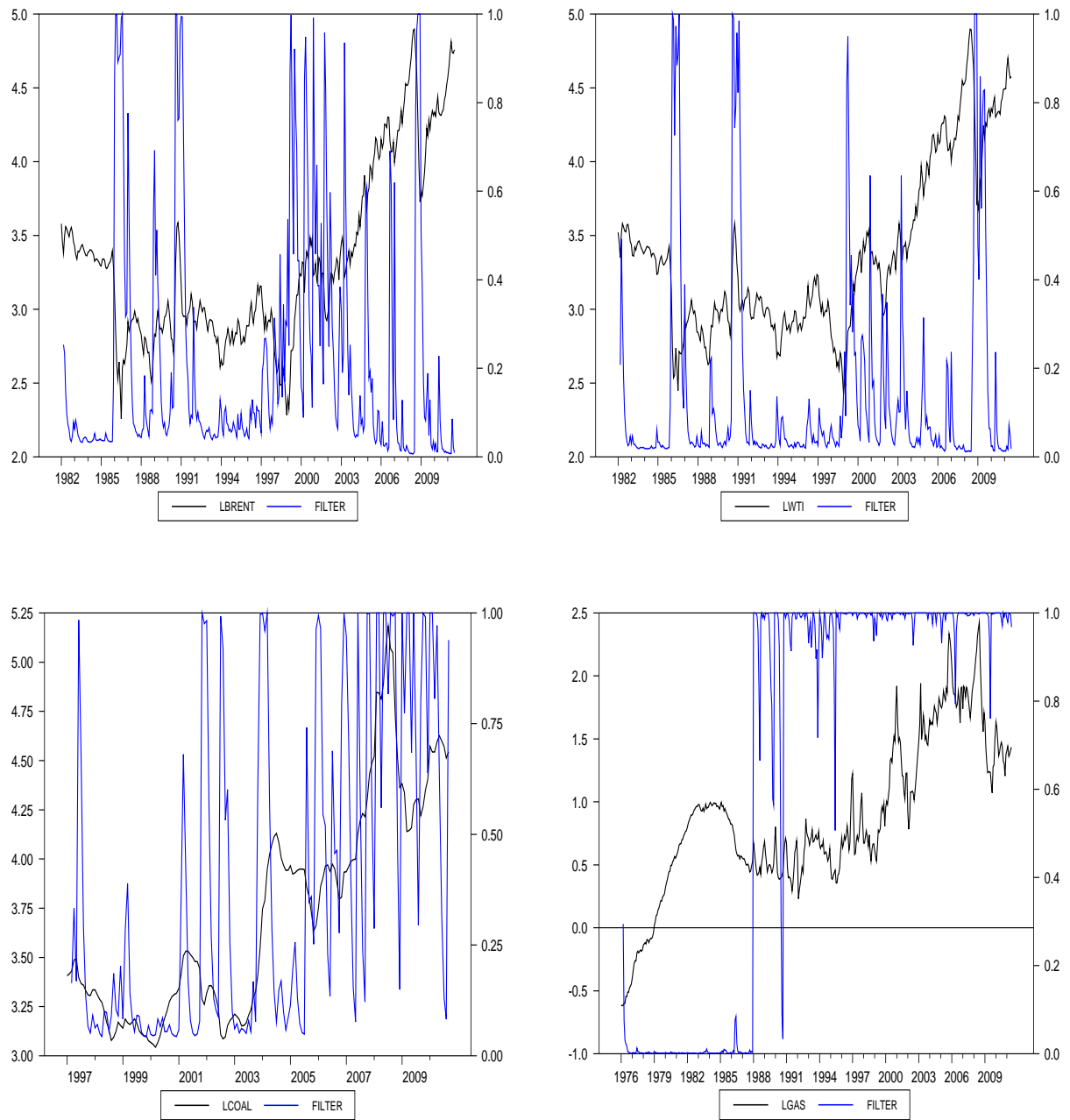


Figure 1: Logarithm of the Brent oil price, WTI oil price, bituminous coal price and natural gas price (black lines) and filtered probabilities for the local non-stationary with high-volatility regime (blue line).