DEVELOPMENT OF A BOTTOM-UP BASED CGE MODEL AND EVALUATION OF ENERGY POLICY

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Overview

This study shows a mathematical formulation of a bottom-up based Computable General Equilibrium (CGE) Model, and a calculated result about the economic impact of nuclear power phase-out. CGE is a class of economic model and has been widely adopted for estimating the effectiveness of environmental policies. CGE models, however, are usually formulated as a top-down model with aggregated production functions, and the homogeneous analytical method is applied to all economic sectors. This conventional CGE model, thus, are not capable of handling detailed engineering process factors. Therefore, the authors develop a bottom-up CGE Model that considers both a general equilibrium of economy and specific technological constraints rooted in each economic sector. The model also considers multiregional global trade without Armington Assumption corresponding to GTAP (Global Trade Analysis Project) 7 Data Base [1] as well. The solution of the model is obtained by solving a convex quadratic programming problem while conventional CGE is usually formulated as non-linear equations.

1. Introduction

There are mainly two types of approach for the formulation of energy model, one is so called bottom-up approach, and the other is top-down approach. The former one, bottom-up approach, is developed by deductive formulations, which optimize certain engineering process, for instance, power generation mix or petroleum refining, to minimize the total system cost. Bottom-up models are able to consider detailed engineering process factor including uninstalled innovative technology. In addition, its result is relatively easy to understand what technology and how capacity should be installed in the future. However, the optimum solution is just a solution of certain energy system, and the solution is not always optimum for whole society. This is an essential limitation of bottom-up models.

On the other hand, the top-down approach is developed by inductive formulation using statistical data. Top-down models are able to treat multiple economic sectors and evaluate propagation to whole economic system. Computable general equilibrium (CGE) model [2] is a typical example of top-down approach. For CGE formulation, optimization behaviour of multiple production activities, household, and other economic sectors are considered to be consistent with actual economic data such as Input-Output table or Social Accounting Matrix. Thus, CGE is an effective way to evaluate how an economy might react to changes in policy since it describes various economic sectors to use actual economic data and it is able to evaluate propagation to whole economy. However, CGE are not capable of treating detailed engineering process because same analytical method is applied to all sectors with aggregated production function. Even mass balance is not always conserved due to modelling of composite commodities.

The both type of energy economic model have essential limitation in terms of modelling actual economy and energy market. Although a bottom-up model is effective to evaluate certain energy system and specific technology, the model is not capable of evaluating propagation to whole economy. By contrast, top-down model is advantageous to evaluate propagation to whole economy, however, the model is unable to handle detailed engineering process and it is difficult to evaluate the economic effect of the deployment of certain technology. Hence, in order to overcome the limitation and obtain consistent solution, we need to develop a new type of energy economic model that considers the essences of both top-down and bottom-up models.

On these backgrounds, the objective of this study is to develop a bottom-up CGE model that considers both detailed engineering process and propagation to whole economy to overcome the limitation of conventional models.

2. Formulation of Bottom-up Based CGE

Descriptions of exogenous and endogenous variables in bottom-up CGE model are shown in Table 1 and Table 2. The variables which are not defined in those tables are defined in context whenever it is used. The bottom-up based CGE model is an originally formulated CGE centering on production activities to consider detailed engineering process. Unlike conventional CGE, the model does not adopt the production function which considers substitution of input commodities. Instead, it explicitly selects specific technology to disaggregate production activities to detailed engineering processes which are also defined as a kind of activities. In this study, we develop a static model which treats input factor, namely, capital and labour, as exogenous variables. It is quite difficult to treat capital consistently in a static model because capital is determined dynamically in the real economy. In addition, it is not always suitable to optimise amount of intermediate input and input factor simultaneously for the model which deeply considers engineering processes. Therefore, the optimum behaviour of each production activity is formulated as maximization of value added as explained in section 2.1 though CGE usually assumes maximization of each profit.

Trades among multiple regions are also considered in the model as shown in section2.2. Its formulation expresses cross-hauling of commodity without Armington Assumption. In this study, although the model considers only 3 regions, it is expected to be modelled as a dynamic model for all over the world. For this reason, GTAP 7 data base, which is available for 113 regions and 57 commodities, is used for equilibrium data in the model.

The model is formulated as a convex quadratic problem subject to linear constraints as explained in section2.3 and 2.4, while conventional CGE usually described as simultaneous non-linear equations. Engineering factor is described by linear equations and disaggregation of activities. This formulation is intended to make it easy to solve a huge problem which has a large number of variables because the model is supposed to have large number of variables and constraints to consider each engineering factor and multiple regions.

Table 1 Exogenous Variables

Name	Dimension	Description
i	-	index of commodity
j	-	index of production activity
r, s	-	index of region
N	-	number of regions
n	-	number of commodities
m	-	number of production activities
l	-	number of constraints.
$\boldsymbol{a}_{j,r}$	$nN\times1$	input coefficient vector of jth activity in region r
$\boldsymbol{a}_{i,r,s}$	$nN\times1$	input coefficient vector of importing activity from r to s with commodity i
$\boldsymbol{b}_{j,r}$	$nN\times1$	$\equiv \boldsymbol{e}_{j,r} - \boldsymbol{a}_{j,r}$
$\boldsymbol{b}_{i,r,s}$	$nN\times1$	$\equiv \boldsymbol{e}_{i,s} - \boldsymbol{a}_{i,r,s}$
$\boldsymbol{c}_{j,r}$	$l\times1$	coefficient vector of linear constraints at variable $x_{j,r}$
$\boldsymbol{c}_{i,r,s}$	$l\times1$	coefficient vector of linear constraints at variable $x_{i,r,s}$
$\boldsymbol{C}$	$l\times(mN + nN^2)$	coefficient matrix of describing all linear constraints
$\boldsymbol{e}_{i,r}$	$nN\times1$	normal vector corresponding to ith commodity in region r
$\boldsymbol{F}$	$nN\times nN$	price strategy matrix
$\boldsymbol{g}_{r}$	$nN\times1$	government consumption vector in region r
$\boldsymbol{i}_{r}$	$nN\times1$	investment vector in region r
$\boldsymbol{k}$	$l\times1$	constant vector for describing constraints
$\boldsymbol{\Delta}_{i,r,s}$	$nN\times nN$	correction matrix for price change of importing activity from r to s
$u_{j,r}$	-	$\equiv \boldsymbol{b}_{j,r}^{T}\boldsymbol{F}^{T}\boldsymbol{b}_{j,r}$
$v_{i,r,s}$	-	$\equiv \boldsymbol{b}_{i,r,s}^{T}\left(\boldsymbol{F}+\boldsymbol{\Delta}_{i,r,s}\right)^{T}\boldsymbol{b}_{i,r,s}$
$\alpha_{i,r}$	-	parameter of utility function U_r
γ_r	-	coefficient of utility function U_r

Table 2 Endogenous Variables

Name	Dimension	Description
$\boldsymbol{x}$	$(mN + nN^2) \times 1$	$\equiv \left(x_{1,1} \ \ldots \ x_{j,r} \ \ldots \ x_{m,N} \ \ x_{1,1,1} \ \ldots \ x_{i,r,s} \ \ldots \ x_{n,N,N}\right)^T$
$\boldsymbol{h_r}$	$nN\times1$	household consumption vector in region *r*
$\boldsymbol{\delta p_{j,r}}$	$nN\times1$	price change by the effect of *j*th production activity
$\boldsymbol{\delta p_{i,r,s}}$	$nN\times1$	price change by the effect of importing activity from *r* to *s*
$\boldsymbol{\lambda}$	$nN\times1$	price vector for all commodities
$\boldsymbol{\mu}$	$l\times1$	shadow price for all constraints
Obj	-	objective function of the model
U_r	-	utility function of household in region *r*
$VA_{j,r}$	-	value added of *j*th in region *r*
$VA_{i,r,s}$	-	value added of importing activity from *r* to *s* with commodity *i*
$x_{j,r}$	-	amount of production by *j*th production activity
$x_{i,r,s}$	-	amount of import by importing activity from *r* to *s*
$h_{i,r}$	-	household consumption of commodity *i* in region *r*
$\lambda_{i,r}$	-	price of commodity *i* in region *r*

2.1. Optimum Behaviour of Domestic Production Activities under Price Strategy

In this section, behaviour of domestic production activities and its formulation are explained. We discuss only domestic activities in specific region, that is $N = 1$ at the Table 1 and Table 2. Therefore the index of region *r* is abbreviated in this section. The optimum behaviour of production activities is maximization of their value added. Value added of *j*th activity is written as follows;

$$VA_j(x_j) = \lambda_j x_j - \boldsymbol{\lambda^T a_j} \cdot x_j = \boldsymbol{\lambda^T b_j} \cdot x_j \tag{1}$$

where $\boldsymbol{\lambda^T}$ means transpose of price vector $\boldsymbol{\lambda}$. Now, we assume that each production activity estimate price change in the market by the effect of its behaviour. Two kind of price change can be considered. One is by the effect of purchase of intermediate commodities, and the other is by the effect of sell of produced commodity. In this model, to simplify the formulation, these two price changes are treated in the same method based on subjective price elasticity. Subjective elasticity $\epsilon_{i_1 i_2}$ is defined as follows;

$$\epsilon_{i_1 i_2} \equiv \frac{\delta \ln d_{i_1}}{\delta \ln p_{i_2}} = \left(\frac{\delta d_{i_1}}{d_{i_1}}\right) / \left(\frac{\delta p_{i_2}}{p_{i_2}}\right) \tag{2}$$

where d_{i_1} is a demand of commodity i_1 for certain production activity, p_{i_2} is a price of commodityi_2. For d_{i_1}, intermediate purchase is treated as positive demand, and sell of produced commodity is treated as negative demand. By the definition of $\epsilon_{i_1 i_2}$, following formula is derived;

$$\begin{pmatrix} \delta d_1 \\ \delta d_2 \\ \vdots \\ \delta d_n \end{pmatrix} = \begin{pmatrix} \frac{d_1}{p_1}\epsilon_{11} & \frac{d_1}{p_2}\epsilon_{12} & \cdots & \frac{d_1}{p_n}\epsilon_{1n} \\ \frac{d_2}{p_1}\epsilon_{21} & \frac{d_2}{p_2}\epsilon_{22} & & \vdots \\ \vdots & & \ddots & \\ \frac{d_n}{p_1}\epsilon_{n1} & \cdots & & \frac{d_n}{p_n}\epsilon_{nn} \end{pmatrix} \begin{pmatrix} \delta p_1 \\ \delta p_2 \\ \vdots \\ \delta p_n \end{pmatrix} \tag{3}$$

Assuming that following inverse matrix exists;

$$\boldsymbol{F} \equiv \begin{pmatrix} \frac{d_1}{p_1}\epsilon_{11} & \frac{d_1}{p_2}\epsilon_{12} & \cdots & \frac{d_1}{p_n}\epsilon_{1n} \\ \frac{d_2}{p_1}\epsilon_{21} & \frac{d_2}{p_2}\epsilon_{22} & & \vdots \\ \vdots & & \ddots & \\ \frac{d_n}{p_1}\epsilon_{n1} & \cdots & & \frac{d_n}{p_n}\epsilon_{nn} \end{pmatrix}^{-1} \tag{4}$$

Then the price change estimated by *j*th activity when it increases amount of production by δx_j is written as follows;

$$\boldsymbol{\delta p_j} = \boldsymbol{F} \cdot (\boldsymbol{\delta d_j}) = \boldsymbol{F} \cdot (-\boldsymbol{b_j} \cdot \delta x_j) \tag{5}$$

where $\boldsymbol{\delta d_j}$ is change of demand of production activity and is equal to $(-\boldsymbol{b_j} \cdot \delta x_j)$. Purchase of intermediate commodities and sell of product are evaluated by the same matrix $\boldsymbol{F}$ when we formulate the subjective price change for each activity. Despite matrix $\boldsymbol{F}$ should have different value depending on activities, we assume the same matrix element for all domestic activities to simplify the formulation. As explained above, value added of *j*th activity when it increases its production by δx_j is written as follows;

$$VA_j(x_j + \delta x_j) = (\boldsymbol{\lambda} + \boldsymbol{\delta p_j})^{\boldsymbol{T}} \boldsymbol{b_j}(x_j + \delta x_j) \tag{6}$$

Assuming that the optimum behaviour of each activity corresponds to market equilibrium, namely, $\partial(VA_j)/\partial(\delta x_j) = 0$ at $\delta x_j = 0$, following optimum condition is derived using $u_j \equiv \boldsymbol{b_j^T F^T b_j}$;

$$\boldsymbol{\lambda^T b_j} - u_j x_j = 0 \tag{7}$$

For existing activities in base data, u_j can be calibrated as stated in section3.3 and we do not need to estimate all matrix element of $\boldsymbol{F}$. In this case, the assumption which assumes that $\boldsymbol{F}$ does not differ depending on activities does not affect to the result. However, if we consider new technology which does not exist in base data, we have to calibrate matrix element $\boldsymbol{F}$ to calculate u_j of the newly defined activity. In this case, we approximate $\boldsymbol{F}$ as a diagonal matrix and calibrate the diagonal elements. This is the reason why we assume the same $\boldsymbol{F}$ for all domestic activities.

2.2. Optimum Behaviour of Importing Activities

Most CGE apply Armington Assumption [3] to describe interactions between multiple regions. Specifically, they assume that commodities that are produced in different regions are not perfectly substituted each other. The assumption enables smaller modelling, though it is able to obtain realistic solution. For this reason, the assumption has been used in various type of CGE, and its availability has been appreciated. Nevertheless, the assumption often becomes a focus of criticism because of non-conservation of mass and uncertainty of elasticity of substitution. Furthermore, formulation based on Armington Assumption is generally given as non-linear form though the bottom-up CGE is desired to be formulated as a convex quadratic programming problem. As stated above, it is not suitable for this model to apply Armington Assumption.

Instead of applying Armington Assumption, for the bottom-up based CGE, trade is formulated by introducing importing activities for each commodity and transit route. An importing activity purchases a commodity from source region as an intermediate input. Then the activity transports the commodity to destination region to sell it. For this formulation, imported commodities and domestic commodities are perfectly substituted in each market. We assume transportation cost and import or export duty is paid by each importing activity. The optimum behaviour of importing activities is defined as maximization of its value added on price-strategy same as domestic production activities. As a result, amount of trade is determined by optimum behaviour of importing activities. Furthermore, cross-hauling of commodity, that is, the export of the same commodity between two regions to each other, is expressed despite the assumption of perfect substitution.

In what follows, we introduce a concept of International Input-Output table for all regions picked up in the model. Namely, all interactions among the regions are described in single matrix. Therefore, an input coefficient vector of a production activity in region *r* and another production activity in region *s* has the same dimension $nN\times1$ as shown in Table **1**. Basically, each production activity purchases intermediate commodities from domestic market, thus element of input coefficient vector in the rest of regions are zero. The input coefficient of importing activities can be also defined under the concept of international Input-Output table. When an importing activity purchases a certain commodity from source region and transport services, the commodity and transport services can be defined as intermediate input of the activity. Such importing activities and input coefficient vectors can be defined for each regions and route, that is, nN^2 activities in total.

Basically, importing activities are treated in the same way as domestic production activities. However, we need to care the price change estimated by each importing activity. For the case of domestic activities, we only consider the price change inside domestic market. By contrast, for the case of importing activities, we need to consider two price changes within two regions. One is price change in source region by the effect of purchasing a commodity, and the other is price change in destination region by the effect of selling the commodity. To express the both price change by importing activities, the price strategy matrix is extended to $nN \times nN$ matrix which describes price changes for all regions and commodities. Then the price change $\boldsymbol{\delta p_{i,r,s}}$ by the importing activity which imports commodity i from region r to s, is written as follows;

$$\boldsymbol{\delta p_{i,r,s}} = \left(\boldsymbol{F} + \boldsymbol{\Delta_{i,r,s}}\right) \cdot \left(-\boldsymbol{b_{i,r,s}} \cdot \delta x_{i,r,s}\right) \tag{8}$$

where $\boldsymbol{\Delta_{i,r,s}}$ is a matrix to correct the price change. Formula (8) is almost the same form of formula (5), which expresses price change of domestic production activities. There are two reasons to introduce the matrix $\boldsymbol{\Delta_{i,r,s}}$. One is to express the difference of price change between purchase from domestic market and foreign market. Therefore, all matrix element of $\boldsymbol{\Delta_{i,r,s}}$ is zero except the element corresponding to commodity i in region r. The other reason is to adjust parameters to be consistent with optimization condition at base data. The matrix $\boldsymbol{F}$ is determined to satisfy equation (7), which is the optimization condition of domestic production activities, thus optimization condition of importing activities are not always satisfied without $\boldsymbol{\Delta_{i,r,s}}$. Using $\boldsymbol{\delta p_{i,r,s}}$ defined in formula (8), the value added of importing activity when it increase amount of import by $\delta x_{i,r,s}$ is written as follows;

$$VA_{i,r,s}(x_{i,r,s} + \delta x_{i,r,s}) = \left(\boldsymbol{\lambda} + \boldsymbol{\delta p_{i,r,s}}\right)^T \cdot \boldsymbol{b_{i,r,s}} \cdot (x_{i,r,s} + \delta x_{i,r,s}) \tag{9}$$

Assuming that $\partial\left(VA_{i,r,s}\right)/\partial\left(\delta x_{i,r,s}\right) = 0$ at $\delta x_{i,r,s} = 0$, the optimization condition is written as follows using $v_{i,r,s} \equiv \boldsymbol{b}_{i,r,s}^{\boldsymbol{T}}\left(\boldsymbol{F} + \boldsymbol{\Delta}_{r,s}\right)^{\boldsymbol{T}}\ \boldsymbol{b}_{i,r,s}$;

$$\boldsymbol{\lambda^T b_{i,r,s}} - v_{i,r,s} \cdot x_{i,r,s} = 0\ (\forall i, r, s) \tag{10}$$

To compare equation (10) to equation (7), you can find that optimization condition of importing activities is almost the same form with the domestic production activities'.

2.3. Consideration of Constraints

One of the unique points of the bottom-up CGE is that it is able to consider detailed engineering process by linear constraints. Linear constraints are generally written as follows;

$$\boldsymbol{Cx} \le \boldsymbol{k} \tag{11}$$

Applying Kuhn-Tucker conditions, the optimization conditions subject to formula (11) are re-written as follows;

$$\boldsymbol{\lambda^T b_{j,r}} - u_{j,r} \cdot x_{j,r} - \boldsymbol{\mu^T c_{j,r}} = 0\ (\forall j, r) \tag{12}$$

$$\boldsymbol{\lambda^T b_{i,r,s}} - v_{i,r,s} \cdot x_{i,r,s} - \boldsymbol{\mu^T c_{i,r,s}} = 0\ (\forall i, r, s) \tag{13}$$

$$\boldsymbol{\mu^T}(\boldsymbol{Cx} - \boldsymbol{k}) = 0 \tag{14}$$

$$\boldsymbol{\mu} \ge 0 \tag{15}$$

Supply-demand balance is written as follows;

$$\sum_r (\boldsymbol{h_r} + \boldsymbol{g_r} + \boldsymbol{i_r}) = \sum_j \sum_r \boldsymbol{b_{j,r}} \cdot x_{j,r} + \sum_i \sum_r \sum_s \boldsymbol{b_{i,r,s}} \cdot x_{i,r,s} \tag{16}$$

In this model, we assume that the market price $\lambda_{i,r}$ is determined by differentiation of household consumption;

$$\lambda_{i,r} = \frac{\partial U_r(\boldsymbol{h_r})}{\partial h_{i,r}}\ (\forall i, r) \tag{17}$$

We need to care that the market price is separately determined from price strategy of each activity. The solution is obtained by solving simultaneous equations from (11) to (17). However, if we suppose huge number of constraints and variables, it is difficult to obtain the solution of the model when we directly solve the equations because some equation is given by non-linear form. In order to obtain solution easier, we consider following maximization problem given by (18);

$$\text{maximize } Obj = \sum_r U_r(\boldsymbol{h_r}) - \frac{1}{2}\sum_r \sum_j u_{j,r} \cdot (x_{j,r})^2 - \frac{1}{2}\sum_i \sum_r \sum_s v_{i,r,s} \cdot (x_{i,r,s})^2$$

$$\text{subject to} \qquad \boldsymbol{Cx} \leq \boldsymbol{k}, \tag{18}$$

$$\sum_r (\boldsymbol{h_r} + \boldsymbol{g_r} + \boldsymbol{i_r}) = \sum_j \sum_r \boldsymbol{b_{j,r}} \cdot x_{j,r} + \sum_i \sum_r \sum_s \boldsymbol{b_{i,r,s}} \cdot x_{i,r,s}$$

Optimization conditions of problem (18) are exactly the same as equations from (11) to (17). Therefore, we are able to obtain the solution to solve the problem (18) instead of solving simultaneous equations.

2.4. Solution Technique

We obtain the solution of the model to solve the problem given by (18). In what follows, we assume Cobb-Douglas function for the utility function of household $U_r(\boldsymbol{h_r})$;

$$U_r(\boldsymbol{h_r}) \equiv \gamma_r \prod_i h_{i,r}^{\alpha_{i,r}} \tag{19}$$

The utility function given by (19) is known as concave function. In addition, quadratic terms of objective function defined by (18) are obviously concave. Therefore, the objective function is concave because summation of concave function generally derives concave function. In this condition, problem (18) is a convex programming problem subject to linear constraints because the problem is defined as maximization of concave function. Convex programming problem is generally easy to solve. In this study, we solve the problem by sequential quadratic programming approach to approximate the objective function. The second order approximation of household utility function around $\boldsymbol{h_r}^{(k)}$ is given by following formula;

$$U_r^{(k)}(\boldsymbol{h_r}) = U_r(\boldsymbol{h_r}^{(k)}) \cdot \left\{\frac{1}{2}\left[S_1^{(k)}(\boldsymbol{h_r})^2 - S_2^{(k)}(\boldsymbol{h_r})\right] + S_1^{(k)}(\boldsymbol{h_r}) + 1\right\} \tag{20}$$

where $S_1^{(k)}(\boldsymbol{h_r}) = \sum_i \alpha_{i,r}/h_{i,r}^{(k)} \cdot (h_{i,r} - h_{i,r}^{(k)})$, $S_2^{(k)}(\boldsymbol{h_r}) = \sum_i \alpha_{i,r}/\left(h_{i,r}^{(k)}\right)^2 \cdot \left(h_{i,r} - h_{i,r}^{(k)}\right)^2$. Index ($k$) means the solution value of kth step. $U_r^{(k)}(\boldsymbol{h_r})$ is applied to the utility function for (k+1)th calculation, then $U_r^{(k)}(\boldsymbol{h_r})$ is updated to $U_r^{(k+1)}(\boldsymbol{h_r})$ based on the solution of (k+1)th step. To iterate this procedure until solution converges, we obtain the solution of original maximization problem.

3. Calibration

3.1. Preparation of Equilibrium Data using GTAP Data Base

In this study, GTAP Data Base is employed for base equilibrium data. GTAP Data Base, which has been developed for Global Trade Analysis Project (GTAP) modelling framework, is the global data base representing the world economy. The bottom-up based CGE model needs bilateral trade data because trade is formulated by the optimum behaviour of importing activities for all routes and commodities. Therefore, GTAP Data Base is suitable to apply for this model. In addition, GTAP Data Base is one of the hugest data base for all regions, hence it is easier to expand the model for detailed disaggregated regions. Consistency of the data is also advantageous to apply it for economic model as is used for GTAP model. The latest version is GTAP 7, which is available for 113 regions and 57 commodities with bilateral trade patterns, production, consumption and intermediate use of commodities and services corresponding to 2004.

GTAP Data Base discriminate domestic commodities and imported commodities for all kind of consumptions including intermediate input and household consumption. However, the bottom-up based CGE assumes the perfect substitution between domestic commodity and imported one since the amount of import is determined by optimum behaviour of importing activity, thus difference between domestic and imported one do not affect to consumers in the model. For this reason, domestic commodity and imported one must be aggregated when we use the data base for the model. For instance, intermediate input of domestic production activities are given by the summation of *VIFM* (*Value of Imports by Firm at Market prices*) and *VDFM* (*Value of Domestic purchases by Firm at Market prices*), and input coefficient is given by division of the summated intermediate input by *VOM* (*Value of Output at Market prices*). Input factor in GTAP, namely, *Land, Unskilled Labour, Skilled Labour, Capital* and *Natural Resource*, are treated as exogenously in the model.

The total amount of transportation service which each importing activity use as intermediate input is assigned to *VTWR* (*Value of Transportation at World prices by Route*). *VTWR* is defined by the difference of CIF and FOB value for each commodity and route. We assume that each importing activity use all type of transportation services, namely, *Water Transport*, *Air Transport* and *Other Transport* in GTAP, by the share of *VST* (*Value of exports of international Transport Services*) to be consistent with the GTAP data. This assumption is not proper for actual importing activities in real economy. For future work, transportation services used by each importing activity are expected to be calibrated properly to be consistent with GTAP data depending on its commodity and route. The input coefficient of importing activities is defined from division of *VXMD* (*Value of eXports at Market prices by Destination*) and each transportation service by *VIMS* (*Value of Imports at Market prices by Source*).

In this study, we evaluate the economic effect of nuclear phase-out in Japan to validate the model formulation, thus, detailed interaction among regions is not required to evaluate. For this reason, regions are aggregate to just 3 regions, Japan, Developed Countries, and Rest of World as shown in Table 3. The classification of Developed Countries has done by OECD high-income countries in 2011 except Japan and the other countries which are not registered in GTAP Data Base. Regarding classification of commodities, we apply original GTAP 57 classifications with no aggregation.

Table 3 Regional Aggregation in the GTAP Data Base

New region	Original GTAP
Japan	Japan
Developed Countries	Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Korea, Luxembourg, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak, Republic, Slovenia, Spain, Sweden, United Kingdom, United States
Rest of World	All Other Regions

3.2. Estimation of Input Coefficient for Each Power Plant

The bottom-up CGE does not allow the substitution of intermediate commodities for domestic production activities. Substitution of commodity means utilization of multiple techniques, for instance, the substitution between LNG and Coal describes the LNG power generation and Coal power generation at electricity sector. Conventional CGE usually assumes these substitutions through formulation of production function for each activity which makes model simpler to calculate because we can simply apply the same type of analytical method to all sectors. However, formulation of aggregated production function is not able to consider specific technique or engineering processes expressly. In addition, it is difficult to understand what technique should be installed or how technical parameter affects the result. For these reasons, the bottom-up CGE does not apply production function which allows substitution of commodities. Instead, we must disaggregate a sector to specific activities which represent each engineering process.

In this study, we disaggregate power generation sector into five kind of activities, namely, Nuclear power, Coal Thermal power, Oil Thermal power, LNG Thermal power, and Hydraulic power. Input coefficient of each power plant is estimated by using Input-Output table in Japan [4], which describes 520 commodities and 407 activities in 2005. Input commodities are aggregated to 57 commodities corresponding to GTAP 7 Data Base. Amount of production for each power plant is estimated to multiply *VOM* at electricity generation sector in GTAP by annual generation ratio of each plant at 2004 [5] [6].

At this time, we have three kinds of data, intermediate input data for each plant estimated from Input-Output table in Japan, amount of production for each plant, and intermediate input data of electricity sector in GTAP Data Base. However, they are inconsistent each other because the data source is different. In this study, we place much value on consistency of GTAP Data Base, thus we adjust the input data estimated by Input-Output table in Japan. Adjustment has done by weighted least square method to minimize value of correction. As a result, the input data of each power plant which is fully consistent with GTAP Data Base is derived for the model. The value of representative commodity for intermediate input is shown in Table 5.

In general, input coefficient must be estimated whenever we consider newly engineering process or technique that is not described in base data. This procedure makes the model difficult to expand in some cases. In addition, the amount of production of each engineering process is treated as endogenous variable. Thus, more we consider detailed engineering factor, the more difficult to obtain solution. This is one of the reasons why we formulate the model as a convex quadratic programming problem.

Table 4 Annual Generation Ratio of each power plant [mill. USD]

Region	Nuclear	Coal Thermal	Oil Thermal	LNG Thermal	Hydro
Japan	40,643	41,808	17,934	36,351	16,826
Developed Countries	164,736	286,755	32,943	130,810	116,941
Rest of World	29,475	215,182	44,651	109,624	112,193

Table 5 Estimated Input Data for each power plant in Japan [mill. USD]

Intermediate Commodity	Nuclear	Coal Thermal	Oil Thermal	LNG Thermal	Hydro
Coal	0	4,153	0	0	0
Oil	0	0	1,415	0	0
Gas	0	0	0	7,001	0
Paper products, publishing	155	194	73	164	44
Petroleum, coal products	13	1,184	4,992	346	0
Chemical,rubber,plastic prods	0	49	17	41	0
Mineral products nec	0	19	4	15	12
Metals nec	57	10	4	9	8
Metal products	60	45	14	37	21
Electronic equipment	9	25	5	19	1
Machinery and equipment nec	8	18	4	14	1
Manufactures nec	59	1,319	286	1,016	27
Electricity	1,771	1,782	771	1,493	554
Construction	1,829	1,855	891	1,571	1,237
Trade	70	750	440	677	24
Transport nec	243	878	438	766	98
Sea transport	1	337	152	293	1
Air transport	15	22	10	20	9
Communication	59	22	213	98	155
Financial services nec	1,152	1,807	820	1,522	524
Insurance	45	10	4	8	18
Business services nec	4,198	6,117	2,751	4,901	1,201
PubAdmin/Defence/Health/Educat	1,256	992	438	846	579

3.3. Calibration of Parameters

This section explains calibration method of γ_r , $u_{j,r}$, $v_{i,r,s}$. At equilibrium data, market prices of all commodities are normalized to one. To consider that market prices are defined by differentiation of utility function of household, following relation is satisfied at base data;

$$\lambda_{i,r}^0 = \frac{\partial U_r(\boldsymbol{h_r^0})}{\partial h_{i,r}^0} = \frac{\alpha_{i,r}}{h_{i,r}^0} \cdot U_r(\boldsymbol{h_r^0}) = 1 \ (\forall i, r) \tag{21}$$

where zero index means the value of base data, for instance, $h_{i,r}^0$ means household consumption of base data. To consider $\alpha_{i,r} = h_{i,r}^0 / \sum_i h_{i,r}^0$, coefficient of utility function γ_r is derived from formula (21) as follows;

$$\gamma_r = \sum_i h_{i,r}^0 / \prod_i h_{i,r}^0 \quad (\forall r) \tag{22}$$

We also need to calibrate $u_{j,r}$ and $v_{i,r,s}$, which are originally defined in the bottom-up based CGE. Although the parameters must satisfy the equation (12) and (13) for base equilibrium data, we are not able to specify its value in one way because $\boldsymbol{\mu}^{\mathbf{0}}$ is unknown at base equilibrium data. The vector $\boldsymbol{\mu}^{\mathbf{0}}$ describes effect of each constraint at base data. If a base data is not constrained by defined inequality shown in (11), the element of $\boldsymbol{\mu}^{\mathbf{0}}$ which corresponds to the inequality must be zero. By contrast, if a base data is constrained by upper or lower limit defined in (11), the element of $\boldsymbol{\mu}^{\mathbf{0}}$ can be positive value. In conventional CGE, $\boldsymbol{\mu}^{\mathbf{0}} = \mathbf{0}$ is assumed implicitly, which means that the equilibrium data is only determined by market and is not affected by technical constraints. When we consider detailed technical factor and apply the bottom-up CGE to evaluate actual energy policy, we need to calibrate the element of $\boldsymbol{\mu}^{\mathbf{0}}$. Although calibration of $\boldsymbol{\mu}^{\mathbf{0}}$ can be sometime arbitrary, we can interpret the base data in various ways, namely, whether amount of production in certain activity is determined by market or technical constraints.

In this study, we simply assume $\boldsymbol{\mu}^{\mathbf{0}} = \mathbf{0}$ for actual calculation as conventional CGE implicitly assumes. In this case, we can specify value of the parameters in one way as follows;

$$u_{j,r} = \frac{(\boldsymbol{\lambda}^{\mathbf{0}})^{\boldsymbol{T}} \boldsymbol{b}_{\boldsymbol{j},\boldsymbol{r}}}{x_{j,r}^{0}} \quad (\forall j, r) \tag{23}$$

$$v_{i,r,s} = \frac{(\boldsymbol{\lambda}^{\mathbf{0}})^{\boldsymbol{T}} \boldsymbol{b}_{\boldsymbol{i},\boldsymbol{r},\boldsymbol{s}}}{x_{i,r,s}^{0}} \quad (\forall i, r, s) \tag{24}$$

Calibration results of $u_{j,r}$ for each plant is shown in Table 6. At formula (23), $(\boldsymbol{\lambda}^{\mathbf{0}})^{\boldsymbol{T}} \boldsymbol{b}_{\boldsymbol{j},\boldsymbol{r}}$ means proportion of value added in each activity. Therefore, if proportion of value added is larger or value of production is smaller, $u_{j,r}$ tends to be large. For this reason, value of $u_{j,r}$ in Japan at fuel production activities such as coal, oil and gas are large because values of production are small. Some sector cannot calibrate $u_{j,r}$ because value of intermediate input is larger than value of production. In this case, $u_{j,r}$ has negative value and the objective function is not concave. To define the objective function as concave, all $u_{j,r}$ and $v_{i,r,s}$ must be positive. Therefore, the amount of production must be fixed for the activities which have negative value of $u_{j,r}$ or $v_{i,r,s}$.

Table 6 Representative Calibration Results of $\boldsymbol{u}_{\boldsymbol{j},\boldsymbol{r}}$ **[**USD^{-1}**]**

Production Activity	Japan	Developed Countries	Rest of World
Nuclear Power	17.92	4.32	22.73
Coal Thermal Power	11.54	1.57	1.73
Oil Thermal Power	12.80	5.47	1.44
LNG Thermal Power	11.70	2.97	2.75
Hydro Power	43.28	6.08	5.97
Coal	13,754	7.52	6.65
Oil	3,028	4.26	1.09
Gas	101,887	10.32	5.40

4. Results

We evaluated the effect of nuclear power phase-out in Japan using the bottom-up based CGE. 57 commodities, 61 production activities and 3 regions were considered in the calculation. To consider technical factor in electricity generation, electricity sector was divided into five kinds of power plant as stated section3.2, Nuclear, Coal Thermal, Oil Thermal, LNG Thermal and Hydro. The upper limit of Hydro power generation and Nuclear power reduction are described as constrains.

We calculated two cases, one is the case of 50% Nuclear power reduction, and the other is the case of 100 % reduction based on the data in 2004 which GTAP 7 corresponds to. Change of annual electricity generation is illustrated in Figure 1. Reduction of Nuclear power is substituted by Thermal powers. Then electricity price was increased by 7% in the case of 50% reduction, 14% in the case of 100% reduction. Electricity demand was decreased following the increase of electricity price as illustrated in Figure 2. Change of GDP value and its breakdown is shown in Table 7 and Table 8 for each case, change ratio is also shown in Table 9 and Table 10. In this calculation, amount of investment and government consumption are treated exogenously, thus change of these variables are equivalent to that of price. The results indicate that Developed Countries and Rest of World increase their export value but change of GDP is very few in ratio. By contrast, Japan decreases its GDP by 0.428% in the case of 50% reduction, 0.854% in the case of 100% reduction. This is mainly due to the decrease in household consumption and increase in import, especially for fossil fuels and manufactures which consume electricity as shown in Table 11.

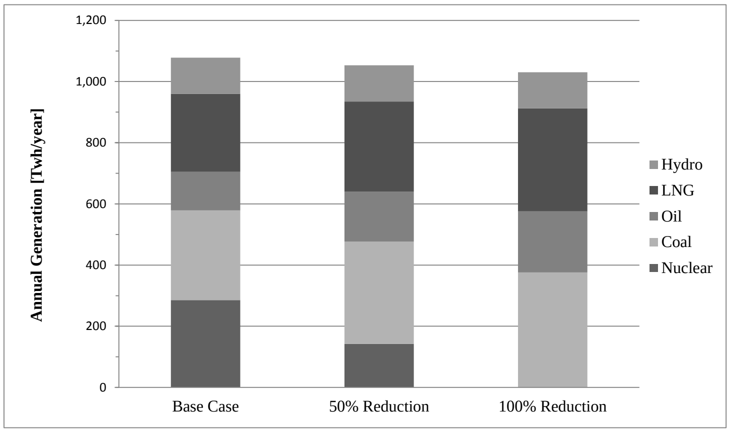

Figure 1 Change of Annual Electricity Generation [TWh/year]

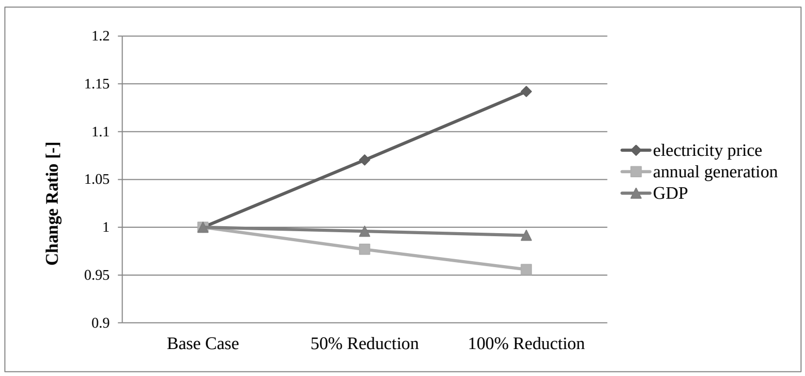

Figure 2 Change Ratios of Electricity Price, Annual Electricity Generation, and GDP [-]

Table 7 Change of GDP in the Case of 50% Nuclear Reduction [mill. USD]

Region	Household Consumption	Investment	Government Consumption	Export	Import	GDP change
Japan	-12,674	-291	-86	-1,081	5,790	-19,922
Developed Countries	-4,192	513	-116	3,759	194	-229
Rest of World	-2,505	338	-34	4,775	1,490	1,083

Table 8 Change of GDP in the Case of 100% Nuclear Reduction [mill. USD]

Region	Household Consumption	Investment	Government Consumption	Export	Import	GDP change
Japan	-25,006	-547	-137	-2,197	11,882	-39,769
Developed Countries	-8,583	1,049	-238	7,658	341	-454
Rest of World	-5,115	688	-69	9,730	3,031	2,203

Table 9 Change of GDP Ratio in the Case of 50% Nuclear Reduction [%]

Region	Household Consumption	Investment	Government Consumption	Export	Import	GDP change
Japan	-0.482%	-0.027%	-0.011%	-0.165%	1.073%	-0.428%
Developed Countries	-0.024%	0.009%	-0.002%	0.060%	0.003%	-0.001%
Rest of World	-0.049%	0.015%	-0.003%	0.133%	0.046%	0.012%

Table 10 Change of GDP Ratio in the Case of 100% Nuclear Reduction [%]

Region	Household Consumption	Investment	Government Consumption	Export	Import	GDP change
Japan	-0.951%	-0.050%	-0.017%	-0.335%	2.203%	-0.854%
Developed Countries	-0.049%	0.019%	-0.005%	0.123%	0.005%	-0.002%
Rest of World	-0.101%	0.031%	-0.006%	0.271%	0.094%	0.025%

Table 11 Representative Change of Import in Japan in the case of 100% Nuclear Reduction

Commodity	Change of Import [mill. USD]	Change Ratio [%]
Coal	1,219	14.42%
Oil	1,237	2.34%
Gas	2,284	20.69%
Paper products, publishing	526	9.80%
Petroleum, coal products	2,134	14.13%
Chemical,rubber,plastic prods	1,521	3.63%
Mineral products nec	828	20.49%
Ferrous metals	1,387	26.88%
Metals nec	708	4.71%
Metal products	310	4.95%
Electronic equipment	685	1.02%
Machinery and equipment nec	693	1.27%
Manufactures nec	407	4.80%

5. Conclusions

This study represents a formulation and solution technique of a bottom-up based CGE which is capable of considering detailed engineering process. As an example of calculation, the economic effect of nuclear phase-out is evaluated using the model. The result shows that reduction of nuclear power decreases GDP in Japan. However, at present, technical factor or engineering process is not described enough to evaluate actual energy policy. In addition, investment is treated as exogenously because the model is formulated as static model. When we evaluate construction of power plant or availability of new technology such as renewable energy, investment cannot be ignored. For future work, it is expected for the model to be extended to a dynamic model which treats investment and capital consistently.

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