

# Development of a Bottom-up based CGE Model and Evaluation of Energy Policy

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**Abstract** — This study shows a mathematical formulation of bottom-up based Computable General Equilibrium (CGE) model and its application for evaluating energy policy. CGE models are a class of economic model and have been widely adopted for estimating the effectiveness of environmental policies. CGE model, however, is not capable of handling detailed factors on engineering process. In this research, therefore, the authors create a bottom-up modeling approach to develop a new CGE model that considers both general equilibrium of economy and the specific technological constraints rooted in each economic sector.

**Keywords** — CGE model, bottom-up approach, economic impact evaluation, input-output tables

## I. INTRODUCTION

We need to investigate the relationship between economy and environment by model analysis to discuss energy policies. There are 2 types of models for economical evaluation, one is so-called top-down based model, and the other is bottom-up based model. Top-down models give us consistent solution from statistical data. However, top-down models are not capable of considering detailed engineering process factors. On the other hand, bottom-up model provides us realistic solution to consider detailed technical factors, but results are not sufficient to describe sectoral economic behavior. Therefore, a model which is developed on the viewpoints from both top-down and bottom-up technique is required for more effective analysis.

Computable General Equilibrium (CGE) model is a class of economic model which have been developed for the evaluation of international trade tariffs or environmental taxation policies. CGE also serves as a popular way to estimate the economic effects of political measures to reduce CO<sub>2</sub> emissions from fossil fuel burning. However, CGE models are usually formulated as a top-down model with aggregated production functions, and the homogeneous analytical method is applied to all economic sectors. Conventional CGE models, thus, are not capable of handling detailed engineering process factors.

## II. OBJECTIVE

An objective of this study is to develop a bottom-up based CGE model considering both general equilibrium of economy and technological constraints in each sector, which allows us to evaluate energy and environmental policy in a more effective way.

In this study, we evaluate energy policy particularly focusing on power generation sector. In our newly

developed CGE model, technological factor and operational constraints of power generation sector are elaborately embedded together with explicitly considering the regulation of CO<sub>2</sub> emissions in this sector. We are capable of evaluating the impact of specific technological factor on whole economy, because the model is constructed from both top-down and bottom-up viewpoints and is expected to enable us to analyze energy and environmental policies in a more effective manner.

## III. METHODS

### A. Formulation of the model

First of all, a static model is presented in this section. Input factor in this study, namely, capital and labor are assumed exogenously as constant value. The model is formulated basically as an optimization problem like conventional CGE models. However, each firm is formulated from bottom up approach, and is supposed to maximize its own value added under imperfect market competition. The value added of  $j$ th firm  $VA_j^0$  is given by:

$$\begin{aligned} VA_j^0 &= \lambda^T \mathbf{e}_j \cdot x_j - \lambda^T \mathbf{a}_j \cdot x_j \\ &= \lambda^T \mathbf{b}_j \cdot x_j \end{aligned} \quad (1)$$

where  $x_j$  is production of  $j$ th firm,  $\lambda$  is price vector,  $\mathbf{e}_j$  is unit vector corresponding to  $j$ th goods,  $\mathbf{a}_j$  is  $j$ th column vector of input coefficient matrix,  $\mathbf{b}_j \equiv \mathbf{e}_j - \mathbf{a}_j$ . We assume that price is obtained by differentiating utility function  $U$  with respect to household consumption  $\mathbf{h}$  as follows:

$$\lambda = \frac{\partial U(\mathbf{h})}{\partial \mathbf{h}} \quad (2)$$

Market competition is formulated to consider strategic behavior of each firm, and the behavior is specifically modeled with parameters of subjective price elasticity. A formulation of the strategy is expressed as:

$$\delta \mathbf{p}_j = \mathbf{F} \cdot \delta \mathbf{d}_j = -\mathbf{F} \mathbf{b}_j \cdot x_j \quad (3)$$

where  $\delta \mathbf{p}$  is amount of price change under the strategy,  $\mathbf{F}$  is strategy matrix based on subjective price elasticity,  $\delta \mathbf{d} \equiv -\mathbf{b}_j \delta x_j$ . The values of the subjective price elasticity of each firm are calibrated so that the resultant optimization behaviors of firms formulated in the bottom up approach should be fully consistent with the national input-output table. When  $j$ th firm increases its production by  $\delta x_j$ , the value added of each firm under the strategy is written as:

$$VA_j = (\lambda + \delta p_j)^T b_j(x_j + \delta x_j) \quad (4)$$

Most CGE models which describe import and export employ Armington assumption. Although Armington assumption is an effective way to develop multiple region models, those formulation are non-linear and mass-balance is not conserved to employ the assumption. In this study, we define importing firms for each goods and region to describe the trading. Assuming that the importing firms behave to maximize its value added under their strategy, we similarly formulate the behavior of importing firms same as domestic firms.

In this model, technological factor is considered with linear inequality constraints given by:

$$Cx \leq k \quad (5)$$

where  $C, k$  are constant matrix or vector, and  $x$  is vector of production variables. Assuming that the strategic behavior of each firm corresponds to the market equilibrium value, optimization condition of each firm is obtained to apply Kuhn-Tucker condition. To discuss optimization condition of each firm, the optimum behavior of each firm can be obtained to solve the optimization problem given by:

$$\begin{aligned} \underset{x}{\text{maximize}} \quad Obj = & \sum_r U_r(\mathbf{h}) - \frac{1}{2} \sum_r \sum_j u_j^r \cdot (x_j^r)^2 \\ & - \frac{1}{2} \sum_{r'} \sum_r \sum_j v_j^{r' \rightarrow r} \cdot (x_j^{r' \rightarrow r})^2 \\ \text{s.t.} \quad Cx \leq & k \\ \mathbf{h} + \mathbf{g} + \mathbf{i} = & \sum_r \sum_j b_{j,r} \cdot x_j^r + \sum_{r'} \sum_r \sum_j b_{j,r' \rightarrow r} \cdot x_j^{r' \rightarrow r} \end{aligned} \quad (6)$$

where  $U_r$  is utility function of household in region  $r$ ,  $x_j^r$  is amount of production of  $j$ th firm in  $r$ ,  $x_j^{r' \rightarrow r}$  is amount of import from  $r'$  to  $r$ ,  $u_j^r$  and  $v_j^{r' \rightarrow r}$  are constant parameters derived from imperfect competition,  $\mathbf{g}$  is government consumption,  $\mathbf{i}$  is investment.

### B. Solution technique

The problem described as (6) is a nonlinear optimization problem with linear inequality constraints. Because the model is supposed to have huge number of variables, it is nontrivial to solve it. In this study, the model is calculated by sequential quadratic programming to approximate the objective function and execute iterative optimization.

## IV. RESULTS

We have developed a model focusing on Japanese domestic economy and evaluated the economic impact through the shutdown of nuclear power plants. The model is derived from Input-Output table 2005 in Japan [1], and is composed of 37 goods and 41 activities, which include power generation of hydro, oil thermal, LNG thermal, coal thermal, and nuclear.

Calculated results are illustrated in Fig 1 and Fig 2. Case 1 is the case of shutdown half of existing nuclear power plants in 2005, Case 2 is the case of shutdown all nuclear plants. As shown in Fig 1 and 2, the model is

capable of evaluating not only the change of power generation mix but also that of GDP and electricity price. However, the results shown in this paper are based on domestic model, thus export and import including fuel for power plant operation is not treated properly in this stage. Currently we are developing a global model that utilizes GTAP (Global Trade Analysis Project) data base, and recalculated results employing the global model will be presented at the conference.

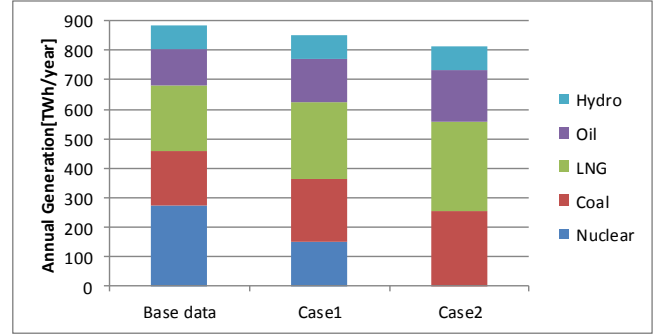


Fig 1. Annual power generation of each plant

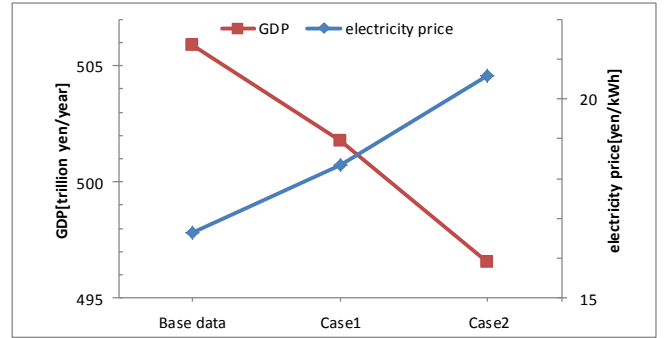


Fig 2. Change of GDP and electricity price

## V. SUMMARY

This study represents a formulation and solution technique of a bottom-up based CGE model which is capable of considering detailed engineering process and constraints in each economic sector. We are currently developing a global model where the engineering factor especially on power generation sector is considered in more detail. For the conference, we will present the evaluation of energy policy regarding nuclear power generation adopting the global model, and discuss the extensions and limitations of the proposed modeling approach.

## REFERENCES

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